MATH 211: 10-16 WORKSHEET

(1) We already saw that if $\rho(t) = \lambda e^{-\lambda t}$, $t \geq 0$, then
$$\int_0^\infty \rho(t)\,\mathrm{d}t = 1.$$
This probability distribution models wait times for repeating random events, with the parameter λ saying how many events happen per time period. For example, if a bus comes three times an hour then $\lambda = 3$ gives a model for how long you'll wait for the next bus to arrive.

Compute the integral
$$\int_0^\infty t\rho(t)\,\mathrm{d}t$$
to confirm that the average wait time is $1/\lambda$.

(2) We saw that the area between $1/x$, $x \geq 1$ and the x-axis is infinite. Rotate this curve around the x-axis and compute the volume of the resulting solid.

(3) Again think about $1/x$, $x \geq 1$ rotated around the x-axis. Calculating its surface area directly is hard, since you would have to integrate
$$\int_0^\infty \frac{2\pi}{x}\sqrt{1 + \frac{1}{x^4}}\,\mathrm{d}x.$$
But you can instead compute a lower bound for the integral, and it'll turn out that that is good enough.

(a) First, explain why $\frac{2\pi}{x}\sqrt{1 + \frac{1}{x^4}} \geq \frac{2\pi}{x}$ for all $x \geq 1$. Then explain why that lets you conclude that
$$\int_0^\infty \frac{2\pi}{x}\sqrt{1 + \frac{1}{x^4}}\,\mathrm{d}x \geq \int_0^\infty \frac{2\pi}{x}\,\mathrm{d}x.$$
(b) Then compute $\int_0^\infty \frac{2\pi}{x}\,\mathrm{d}x$ to get a lower bound for the surface area.

(c) Compare the surface area of the solid to its volume. Whoa.

(4) Calculate the integral
$$\int_0^1 \frac{\mathrm{d}x}{\sqrt{x}}.$$