MATH 211: STUDY GUIDE FOR MIDTERM 1

Here are the big things you should know for the exam.

- How to use integration to determine the area of a region bounded by curves.
- How to use integration to determine the volume of a shape formed by rotating a 2d region around an axis.
- How to use integration to determine arc length of a curve.
- How to use integration to determine the surface area of a shape formed by rotating a 2d region around an axis.
- How to use integration to get information about mass, the first moment, and center of mass if you know about the linear density of an object.
- How to solve problems involving exponential growth and decay.

You should be able to solve the integrals you set up to solve these types of problems. But in practice, there's two obstacles to having that be part of the exam: it might take a long time to solve one integral, or it might not be feasible without computer tools. For that reason you should expect that some—but not all—of the exam problems will only ask you to set up the integral.

If you are asked to set up a formula involving an integral, your final answer should be sufficiently precise that someone could take your answer and plug it into a computer tool to get an approximation. There should be no extra information needed. For example, setting up an integral to compute an arc length of $y = f(x)$ involves the derivative. You should actually calculate the derivative and substitute it into the formula, not just leave it as $f'(x)$.

You will be given the following two formulas:

$$\int_a^b \sqrt{1 + [f'(x)]^2}\,\mathrm{d}x$$

$$\int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,\mathrm{d}x$$

You should know what each of these formulas is computing, how to adjust them to use y instead of x, and so on. For the other formulas you should know, if you can remember the picture of splitting the problem into infinitely small pieces, that picture will give the formula.

Here's a list of sample problems.

(1) The square with corners $(1, 0)$, $(0, 1)$, $(1, 2)$, and $(2, 1)$ is rotated around the x-axis to produce a solid. Set up a formula using integration which gives the volume of this solid.

(2) An apple with its core removed is modeled as a sphere with a cylinder through the center cut away. If the sphere has radius 1.5 inches and the cylinder has radius 0.25 inches, set up a formula using integration which gives the volume of the apple sans core.

(3) The curve $y = xe^x$, for $0 \leq x \leq 1$, is rotated around the y axis to produce a surface. Set up a formula using integration which gives the surface area of this surface.

(4) Set up an integral to determine the length of the curve $y = \ln(\cos x)$, where $-\pi/3 \leq x \leq \pi/3$.

(5) Determine the area of the bounded region bordered by the curves $y = \cos x$ and $y = 4x^2 - \pi^2$.

(6) Determine the area of the bounded region bordered by the curves $y = 0$, $y = x^2$, and $y = 15 - 2x$.

(7) A length of rope has its density given by the function $\rho(x) = 4\sqrt{x}$, where $1 \leq x \leq 4$. Determine the total mass of the rope and its center of mass.

(8) Empirical observations suggest that $\rho(t) = e^{-t} + e^{-10}/10$, where $0 \leq t \leq 10$, is a good model for the probability that your bus arrives t minutes late. Determine the mean amount of time you will expect to wait. [Hint: $\frac{\mathrm{d}}{\mathrm{d}t}(-te^{-t} - e^{-t}) = te^{-t}$.]

(9) Exponential growth is a good model for determining the population of a certain bacteria culture in a lab. You started out with 1000 cells, and after 48 hours have 1650 cells. Write a function which models the population $P(t)$ after t hours, and use this to determine when the population will reach $10,000$ cells.