MATH 210: 9-15 WORKSHEET

Use the limit definition of the derivative to demonstrate why the following facts about derivatives are true.

(1) $\frac{\mathrm{d}}{\mathrm{d}x}x^3 = 3x^2$
(2) $\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x} = -\frac{1}{x^2}$
(3) If $f(x)$ is a function with a derivative and c is a constant, then $\frac{\mathrm{d}}{\mathrm{d}x}cf(x) = cf'(x)$.
(4) If $f(x)$ and $g(x)$ are functions with derivatives then $\frac{\mathrm{d}}{\mathrm{d}x}(f(x)+g(x)) = f'(x)+g'(x)$.
(5) If n is a positive integer then $\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$.
[Hint: remember the binomial theorem.[1]]

[1]Namely,

$$(A+B)^n = A^n + nA^{n-1}B + \binom{n}{n-2}A^{n-2}B^2 + \cdots + \binom{n}{2}A^2B^{n-2} + nAB^{n-1} + B^n,$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the binomial coefficient.