MATH 210: 11-8 WORKSHEET

Theorem (FTC part I)**.** *Suppose $f(x)$ is continuous on $[a,b]$. If*

$$F(x) = \int_a^x f(t)\,\mathrm{d}t$$

then $F(x)$ is an antiderivative of $f(x)$. That is, $F'(x) = f(x)$.

Theorem (FTC part II)**.** *Suppose $f(x)$ is continuous on $[a,b]$ and $F(x)$ is any antiderivative of $f(x)$. Then,*

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a).$$

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(1) Use the FTC part I to find the derivative of

$$F(x) = \int_{-1000}^x e^{-t^2}\,\mathrm{d}t.$$

(2) Use the FTC part I to find the derivative of

$$G(x) = \int_0^{x^2} \sqrt{t}\,\mathrm{d}t.$$

[Hint: if $u = x^2$ then you can compute $G'(u)$ directly. Now use the chain rule.]

(3) It's convenient to talk about integrals where the lower limit is larger than the upper limit. If $a > b$, then define

$$\int_a^b f(x)\,\mathrm{d}x = -\int_b^a f(x)\,\mathrm{d}x.$$

Use this definition to find the derivative of

$$H(x) = \int_x^3 t^4\,\mathrm{d}t.$$

(4) Find the derivative of

$$I(x) = \int_x^{x^3+1} e^t\,\mathrm{d}t.$$

[Hint: split the integral into two pieces, one from x to 0 and the other from 0 to $x^3 + 1$.]

(5) Confirm why the FTC part II is true, knowing part I.
- First, explain why $F(b) - F(a)$ is the same no matter which antiderivative $F(x)$ you look at. [Hint: you need to show that if $F_0(x)$ and $F_1(x)$ are two different antiderivatives then $F_0(b) - F_0(a) = F_1(b) - F_1(a)$. What do you know about how antiderivatives of the same function relate?]
- Next, check that using the antiderivative $F(x)$ given by the FTC part I satisfies the equation in the conclusion of the FTC part II. [Hint: explain why $F(a) = 0$.]
- Conclude that any antiderivative $F(x)$ will satisfy the equatoin in the conclusion of the FTC part II.