MATH 210: 11-27 WORKSHEET

Recall how the *net change* of a function tells you about values of the function:

$$f(b) = \underbrace{f(a)}_{\text{init. val.}} + \underbrace{\int_a^b f'(x)\,\mathrm{d}x}_{\text{net change}}.$$

(1) Suppose the horizontal position of a submarine is given by the function $x(t) = 400t^2$, where t is measured in minutes and $x(t)$ is measured in yards.
   (a) Determine the initial position (its position at $t = 0$) of the submarine.
   (b) Determine the position of the submarine at time $t = 1$ minutes.
   (c) Compute a function for the velocity $v(t)$ of the submarine.
   (d) Compute the net change $\displaystyle\int_0^1 v(t)\,\mathrm{d}t$ of the submarine's position, and compare to what you got in the first two parts. Explain what you see.

(2) You have a lamp with a genie inside, and the genie offers you three wishes. You've been stuck on a calculus problem so you use your wishes to help you calculate $\int_1^4 f(x)\,\mathrm{d}x$ for some fiendishly complicated function $f(x)$:
   - Knowing genies like to twist wishes, your first wish is to avoid this. You wish for the genie to fix a single antiderivative $F(x)$ of $f(x)$ to consider. The genie does so.
   - Your second wish is to know $F(1)$. The genie tells you $F(1) = 10$.
   - Your third wish is to know $F(4)$. The genie, realizing what you want, instead tells you that $\int_1^4 f(x)\,\mathrm{d}x = 90$.

   While that is what you wanted to use $F(1)$ and $F(4)$ to figure out, you still feel cheated. How you can you nonetheless determine $F(4)$?

(3) Suppose you know that the area of a certain shape is increasing at a constant (but unknown) rate. You measure that the area at time $t = 1$ is 12π and the area at time $t = 4$ is 36π. First find the net change of the area of the shape from $t = 1$ to $t = 4$, then use that to determine the rate at which the area is increasing.