MATH 210: 10-16 WORKSHEET

Here's some related rates problems to practice with. For all of these, the key part of the work is to figure out the relationship between the variables you are interested in. Good first steps are to draw a picture, and to identify what rates you are given and what rate you are trying to find.

(1) You have gone back in time to watch the launch of Sputnik, the first human-created satellite to orbit the earth. You position yourself 2 miles from the launch site. The rocket launches straight upward and accelerates until it reaches a constant velocity of 600 miles per hour.
   (a) Write a formula that relates the angle above the horizon you have to face to see the rocket to the current height of the rocket. Use this formula to determine the change in the angle (in radians per hour) when the rocket is 2 miles above the ground.
   (b) Write a formula that relates the distance from you to the rocket and the rocket's current height above the ground. Use this formula to determine how quickly that distance is changing when the rocket is 2 miles above the ground.

(2) After watching the launch of Sputnik, you hop back in your time machine to head to Cafè Landtmann in Vienna to grab a coffee with Sigmund Freud. You are walking north along the street Universitätsring to reach the café, when you spot your friend walking across the park on the opposite side of the street. Freud is walking at a right angle to the street you are walking along. The street is 10 meters wide and you are 20 meters south of the crossing when he reaches a pedestrian crossing on the opposite side of the street. If you are walking at a brisk 5 kilometers per hour, and he is walking at a leisurely 2 kilometers per hour, then how quickly is the distance between you two changing when Freud reaches the pedestrian crossing?

(3) After your Viennese coffee with Freud you get back in the time machine to visit 17th century physicist Robert Hooke. He's excited to tell you about his work on springs. He tells you that, as a consequence of what he proudly calls Hooke's Law,[1] that he has a formula for the potential energy in a compressed spring:

$$E = \frac{1}{2}kx^2,$$

where E is the potential energy, k is a constant that depends on the specific materials and construction of the spring, and x is the distance it has been compressed. Hooke shows you a spring. He tells you he calculated its constant as $k = 0.4$ Joules per square centimeter. And to demonstrate he begins to compress the spring at a constant rate of 0.5 centimeters per second.

Work out a formula for the change $\frac{\mathrm{d}E}{\mathrm{d}t}$ in the energy in terms of k, x, and $\frac{\mathrm{d}x}{\mathrm{d}t}$. Use this formula to determine the change in the potential energy when the spring has been compressed by 8 centimeters.

[1] `https://en.wikipedia.org/wiki/Hooke's_law`