MATH 210: 10-11 WORKSHEET

If you have an equation which describes a curve, e.g. $x^2 + y^2 = 1$, then *implicit differentiation* can give you a formula for the slope of the curve. Namely, you differentiate both sides with respect to x, treating y as a function of x and using the chain rule. For the circle example, you would get $2x + 2y \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = 0$, and you can solve $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y}$.

(1) A circle of radius r is given by the equation $x^2+y^2 = r^2$. Use implicit differentiation to get a formula for the slope, and use it to determine the slope at the point $(r/2, r\sqrt{3}/2)$. More generally, what is the slope at the point $(r\cos\theta, r\sin\theta)$?

(2) To compare with other methods: Solve for y in terms of x in the equation $x^2 + y^2 = r^2$ to get two functions which describe the top and bottom halves of the circle. Differentiate these functions to get formulas for the slope at an x-coordinate x. Check that these formulas are equivalent to what you get from implicit differentiation.

(3) An *ellipse* is like a circle that's been stretched in one direction and is given by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where $a, b > 0$. Use implicit differentiation to get a formula for the slope. Check that when $a = b = r$ this gives the same formula for slope as with a circle of radius r.

(4) A *hyperbola* facing up-down is given by the equation

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where $a, b > 0$. Get a formula for the slope. Determine all intercepts of the hyperbola and the slopes at those intercepts.

(5) Consider the hyperbola $x^2 - y^2 = 16$. Determine the slope of the the hyperbola at the point $(5, -3)$.

(6) The equation $2x^3 + 2y^3 - 9xy = 0$ describes a curve. Determine a formula for the slope of the curve at the point (x, y). Determine the tangent line to the curve at the point $(2, 1)$. Use computer tools to graph the curve and line to check your calculation.