MATH 210: STUDY GUIDE FOR MIDTERM 1

Here are the big things you should know for the exam.

- The rules for derivatives.
  You should know all of the rules on the handout and how to apply them to compute derivatives. **You will not have a formula sheet for the exam.**
- Formal understanding of calculus concepts.
  You should understand the formal definitions of the derivative and continuity, and how to apply them. You should know the rules for calculating limits.
- Intuitive understanding of calculus concepts.
  You should understand the geometric meaning of limits, continuity, and derivatives. You should know how to get information about these and about differentiability by looking at a graph. You should understand what the Intermediate Value Theorem says and how to check when it applies.

Here's a list of sample problems. If you can do them without referring to notes, a calculator, or a formula sheet then you should feel prepared for the exam. [Please note that the study guide is longer than the actual midterm.]

(1) Differentiate $a(x) = e^x \cos x$. What is $a'(0)$?
(2) Find the first and second derivatives of $b(t) = 5000 + t - 10t^5$.
(3) Differentiate $c(x) = \sqrt{1 + \ln x}$. What is $c'(1)$?
(4) Differentiate $d(x) = \arcsin(\tan(2x))$.
(5) Differentiate $f(x) = \frac{e^{x^2+e}}{x^2+e}$.
(6) Differentiate $g(x) = \arctan(x + \csc x)$.
(7) Calculate the limit

$$\lim_{x\to 1} \frac{(x-1)\left(\sqrt{x+3}-2\right)}{x^2-2x+1}.$$

(8) Calculate the limit

$$\lim_{x\to 3} \frac{x^2-2x-3}{x^3-3x^2}.$$

(9) Calculate the limit

$$\lim_{x\to\infty} \frac{x^2-2x-3}{x^3-3x^2}.$$

(10) Calculate the limit

$$\lim_{x\to 3^+} -3x^3 \ln(x-3).$$

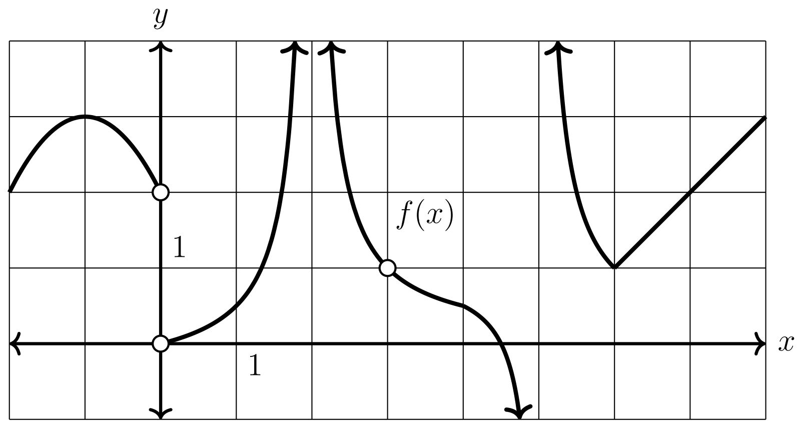
(11) Consider the piecewise-defined function

$$w(x) = \begin{cases} 3-2x & \text{if } x < 1 \\ x^2 & \text{if } 1 < x < 4 \\ \log_2(x) & \text{if } 4 < x \end{cases}$$

Find all discontinuities of $w(x)$ and classify them.
(12) Use the other rules for derivatives to derive the rules for $\tan x$ and $\sec x$.
(13) Use the limit definition of the derivative to compute the derivative of $x^2 - x$.

(14) Use the limit definition of the derivative to compute the derivative of $1/\sqrt{x}$.

(15) The function $f(x)$ is given by the below graph. Use the graph to determine the following limits. (Write "DNE" if the limit does not exist.)



$\lim_{x\to 0^-} f(x)$	$\lim_{x\to 0^+} f(x)$	$\lim_{x\to 0} f(x)$
$\lim_{x\to 2} f(x)$	$\lim_{x\to 3} f(x)$	$\lim_{x\to 5^-} f(x)$
$\lim_{x\to 5^+} f(x)$	$\lim_{x\to 5} f(x)$	$\lim_{x\to 7} f(x)$

(16) Using the same graph, what are the x-coordinates for the discontinuities of $f(x)$? Classify each discontinuity.

(17) Using the same graph, identify all x values where $f(x)$ isn't differentiable. For each one, explain why the function isn't differentiable there.

(18) Is there a function $f(x)$ so that $f(0) = 3$, $f(4) = 0$, but there is no input x with $f(x) = 1$? If yes, justify your answer with an example. If no, justify your answer with an explanation.

(19) Your friend claims that if a function $f(x)$ has both positive and negative slopes somewhere then there must be a point x so that $f'(x) = 0$. Their argument goes as follows:

- Let a be a point where $f'(a) < 0$ and b be a point where $f'(b) > 0$. We can find these points because that's what it means to have negative and positive slope somewhere.
- $f'(x)$ is a continuous function, so by the intermediate value theorem there is a point x between a and b where $f'(x) = 0$.

Identify the fallacious step in your friend's argument and explain why it doesn't work. Then follow up by producing a counterexample to their claim.