Math 210 Midterm 2

Name: Answer Key

This is the midterm for unit 2.

Carefully read each question and understand what is being asked before you start to solve the problem. Please show your work in an orderly fashion, and circle or mark in some way your final answers.

No calculators nor other electronic devices are allowed.

1. (10 points) (a) Evaluate the indefinite integral

$$\int 2\sin x + (2x)^3\,dx.$$

$= 8x^3$

$$= -2\cos x + 2x^4 + C$$

2 2 1

(a) Evaluate the definite integral

$$\int_0^{\pi} 2\sin x + (2x)^3\,dx.$$

$$= \left(-2\cos x + 2x^4 \Big|_0^{\pi}\right] \quad 2$$

$$= (-2\cos\pi + 2\pi^4) - (-2\cos 0 + 0)$$

1 1 1

$$= 2 + 2\pi^4 + 2$$

$$= 4 + 2\pi^4$$

Seach

2. (10 points) (*a*) Evaluate the limit

$$\lim_{t\to\infty} \frac{t\ln t}{t^2+1}$$ $\frac{\infty}{\infty}$

$= \lim_{t\to\infty} \frac{\ln t + 1}{2t} = \lim_{t\to\infty} \frac{1/t}{2} = 0$

$\frac{\infty}{\infty}$

(*b*) Evaluate the limit

$$\lim_{x\to 0} x\cot x.$$ $0\cdot\infty$

$= \lim_{x\to 0} \frac{x}{\tan x} = \lim_{x\to 0} \frac{1}{\sec^2 x} = \lim_{x\to 0} \cos^2 x = 1$

3. (10 points) Find the first derivative of $y(x) = (x-1)^{x^2-1}$.

4 $\ln y = (x^2-1)\ln(x-1)$

6 $\frac{y'}{y} = \frac{x^2-1}{x-1} + 2x\ln(x-1)$

$y' = (x-1)^{x^2-1} \cdot (x+1 + 2x\ln(x-1))$

−8 if try power rule, no log. diff.

4. (10 points) **Do exactly one of the following. If you attempt both, cross out the one you don't wish me to grade.**

(*a*) Use the rule for the derivative of e^x and implicit differentiation to derive the differentiation rule for $\ln x$.

$y = \ln x$

$e^y = x$

$e^y \cdot y' = 1$

$y' = \frac{1}{e^y}$

$\boxed{y' = \frac{1}{x}}$

(*b*) Use the rule for the derivative of $\ln x$ and logarithmic differentiation to derive the differentiation rule for e^x.

$y = e^x$

$\ln y = \ln(e^x)$

$= x$

$\frac{y'}{y} = 1$

$y' = y$

$\boxed{y' = e^x}$

5. (10 points) A cube is shrinking with the volume changing decreasing at a constant rate of 10 cubic feet per minute. How quickly is the surface area of the cube changing when the volume is 1000 cubic foot? **Give an exact answer.**

Here are the formulas for volume and surface area of a cube, based on the length x of a side:

$$V = x^3 \qquad A = 6x^2$$

$\frac{dV}{dt} = -10\ Ft^3/min$ what is $\frac{dA}{dt}$ when $V = 1000\ Ft^3$

$V = x^3 \Rightarrow x = \sqrt[3]{V}$

$A = 6x^2 \Rightarrow A = 6 \cdot V^{2/3}$

$\frac{dA}{dt} = 6 \cdot \frac{2}{3} \cdot V^{-1/3} \cdot \frac{dV}{dt} = \frac{4 \cdot dV/dt}{\sqrt[3]{V}}$

@ $V = 1000\ Ft^3$:

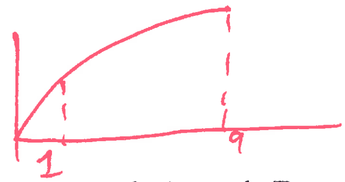
$\frac{dA}{dt} = \frac{4 \cdot (-10)}{10} \quad \frac{Ft^3/min}{Ft}$

$= -4\ Ft^2/min$

A decreasing at 4 Ft^2/min.

−6, if forget chain rule

6. (10 points) Consider the definite integral $\int_1^9 \sqrt{x}\,dx$.



(a) Set up the left Riemann sum with $N = 8$ regions to approximate the integral. **Do not calculate the sum, just set it up. Your summand should only depend on the index variable, so that you could give it to someone who doesn't know the context and they'd be able to calculate it.**

$\underbrace{\Delta x = \frac{9-1}{8} = 1}_{1}$ $\qquad$ $\underbrace{x_i = 1+i}_{1}$

$$\sum_{i=0}^{7} \underbrace{\boxed{\sqrt{1+i}\cdot 1}}_{1}$$

1 1

(b) Set up the right Riemann sum with $N = 4$ regions to approximate the integral. **Do not calculate the sum, just set it up. Your summand should only depend on the index variable, so that you could give it to someone who doesn't know the context and they'd be able to calculate it.**

$\Delta x = \frac{9-1}{4} = 2 \qquad x_i = 1+2i$

$$\sum_{i=1}^{4} \sqrt{1+2i}\cdot 2$$

7. (10 points) What is the slope of the curve $x^2 - 3xy + y^2 = 1$ at each of its x-intercepts?

2 x-ints: $y=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1$

at $(\pm 1, 0)$

6 slope: $2x - 3xy' - 3y + 2y \cdot y' = 0$

$y'(2y-3x) = 3y-2x$

$y' = \dfrac{3y-2x}{2y-3x}$

at $(1,0)$: $y' = \dfrac{-2}{-3} = \dfrac{2}{3}$

2 at $(-1,0)$ $y' = \dfrac{2}{3}$

slope is $\frac{2}{3}$ at each x-intercept

8. (15 points) Consider the function $w(x) = x^9 e^{-x}$. Determine the intervals on which $w(x)$ is increasing and on which it is decreasing. Find where all local maximums and minimums of $w(x)$ are at. Give your answers for the intervals in interval notation. If for any of these there are none, just write "none".

3

- Increasing on $(-\infty, 0) \cup (0, 9)$
- Decreasing on $(9, \infty)$
- Local maximum(s) at $x =$ 9
- Local minimum(s) at $x =$ None

5 $\quad w'(x) = e^{-x}(-x^9 + 9x^8)$

4 $\quad w'(x) = 0 \iff -x^9 + 9x^8 = 0$ (b/c e^{-x} never $= 0$)

$-x^8(x-9) = 0$

$x = 0$ or $x = 9$

$w'(x)$ 3: $+$ | 0 | $+$ | 9 | $-$

$w(x)$: incr | incr | decr

$x=1$: $w'(1) = \frac{1}{e}(-1+9) > 0$

$x=-1$ $\quad w'(-1) = e(1+9) > 0$

$x=10$: $w'(10) = e^{10}(-10^9 + 9 \cdot 10^8) < 0$

9. (15 points) Minimize the quantity $\frac{4}{x+1} - y$ subject to the constraint that x and y are both nonnegative and sum to 3. What values for x and y give the minimum? What is the value of the minimum?

- $x =$ 1
- $y =$ 2
- Minimum is 0

2

$x+y=3$ $\Downarrow$ $y=3-x$

$x, y \geq 0$ so $x \leq 3$

$v(x) = \frac{4}{x+1} - (3-x) = \frac{4}{x+1} + x - 3$

minimize on $[0,3]$

5

$$\boxed{v'(x) = \frac{-4}{(x+1)^2} + 1}$$

4

$v'(x)=0$ when $1 = \frac{4}{(x+1)^2}$

$(x+1)^2 = 4$

$x+1 = \pm 2$

$x = -1 \pm 2$

$x = -3, 1$

not relevant

2

possible place for min

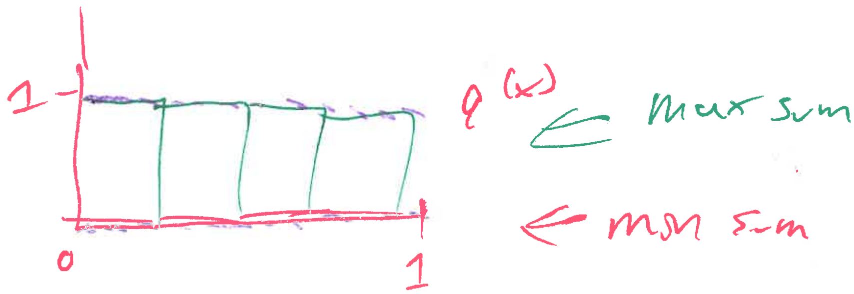
x	$v(x)$	
0	1	
1	0	min
3	1	

2

10. **Extra Credit** (Up to +5) Consider the function

$$q(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$

Is $q(x)$ integrable on $[0, 1]$? If yes, calculate the integral $\int_0^1 q(x)\, dx$. If no, explain why.



The max Riemann sum is always 1 no matter what N is cuz all rectangles have height 1. Similarly, the min Riemann sum is always 0.

So in the limit, you don't always get the same value regardless of your method for picking where to measure the height.

So $q(x)$ is <u>not</u> integrable on $[0,1]$.

$\int_0^1 q(x)dx$ doesn't make sense.