

**MATH 1420: WORKSHEET FOR SECTION 4.7**  
**OPTIMIZATION**

**Problem 1.** Find the positive number  $x$  which minimizes the sum of the number and its reciprocal.

**Problem 2.** Find two numbers  $x$  and  $y$  between 0 and 10 so that the sum of their squares is maximized. Then find the two numbers so that the sum of their squares is minimized.

**Problem 3.** An ellipse is given by the equation

$$\frac{x^2}{9} + y^2 = 1.$$

You want to inscribe a rectangle inside the ellipse, so that the corners of the rectangle are on the ellipse. What should the coordinates of the corners be to maximize the area? What is this maximum area? [Hint: first graph the ellipse with a graphing calculator. The rectangle should have sides parallel to the axes.]

**Problem 4.** Which point on the line  $y = 5 - 2x$  is closest to the origin? How far is it from the origin? [Hint: the distance of a point  $(x, y)$  to the origin  $(0, 0)$  is given by  $\sqrt{x^2 + y^2}$ .]

**Problem 5.** Which point on the parabola  $y = -x^2$  is closest to the point  $(0, -2)$ ? [Hint: the distance between two points  $(x_0, y_0)$  and  $(x_1, y_1)$  is  $\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$ .]

**Problem 6.** You are constructing a topless box for your cat to sleep in. The plush material for the bottom costs 5 dollars per square foot, while the material for the sides is cheaper at 3 dollars per square foot. You need a box which fills a volume of 6 cubic feet. Determine the dimensions of the box that would minimize the cost, then build a larger box anyway because your cat deserves luxury.

**Problem 7.** Two perfectly vertical poles are to be connected by a wire that is also anchored to the ground between them, so that the wire forms a V shape. One pole is 40 feet tall, while the other pole is 30 feet tall. If the distance between the poles is 60 feet and the ground is perfectly flat between them, where should the wire be anchored to the ground to minimize the length of wire needed?