In-Class Questions: March 23, 2023

1. Determine whether **Rolle's Theorem** applies to the given function on the specified interval $[a, b]$. If Rolle's Theorem applies, find all c in the open interval (a, b) which satisfies the conclusion of the theorem. If Rolle's Theorem does not apply, write "Rolle's Theorem does not apply". Provide justification for your conclusions.

(a) $x^2 - 3x - 16$ on $[-2, 5]$

(b) $2x^3 - 9x^2$ on $[-2, 2]$

(c) $3x^2 - 2x - 1$ on $[-1, 1]$

(d) $\dfrac{x^2}{x^2 - 16}$ on $[-5, 5]$

(e) $\dfrac{x^2 - 2x + 1}{x^2 - 2x - 24}$ on $[0, 2]$

(f) $|x - 4|$ on $[0, 2]$

(g) $|x^2 - 4|$ on $[-2, 2]$

2. Determine whether the **Mean Value Theorem (MVT)** applies to the given function on the specified interval $[a, b]$. If MVT applies, find all c in the open interval (a, b) which satisfies the conclusion of the theorem. If MVT does not apply, write "MVT does not apply". Provide justification for your conclusions.

(a) $x^2 + 3x - 4$ on $[0, 2]$

(b) $x^2 + 8x + 12$ on $[-1, 1]$

(c) $-\dfrac{x^2}{x^2 - 4}$ on $[-3, 3]$

(d) $3x^2 + 2x - 2$ on $[-1, 0]$

(e) $|x^2 - x - 2|$ on $[-1, 3]$

(f) $|x^2 - x - 12|$ on $[-3, 4]$

(g) $\sqrt{16 + x^2}$ on $[0, 3]$

3. Use the Mean Value Theorem to answer each of the following.

(a) If $f(6) = 2$ and $1 \le f'(x) < \infty$ for all x, what is the smallest value of $f(7)$?

(b) If $f(2) = -3$ and $-4 \le f'(x) < \infty$ for all x, what is the smallest value of $f(8)$?

(c) If $f(-1) = 0$ and $-\infty < f'(x) \le 5$ for all x, what is the largest value of $f(0)$?

(d) If $f(3) = 5$ and $-\infty < f'(x) \le -4$ for all x, what is the largest value of $f(6)$?

Theorem 1 (Rolle's Theorem)**.** If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), and if $f(a) = f(b)$, then there is at least one point c in (a, b) for which $f'(c) = 0$.

Theorem 2 (Mean Value Theorem (MVT))**.** If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there is at least one point c in (a, b) for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

A **corollary** is a fact which logically follows from a theorem.

Corollary 1. If $f'(x) = 0$ everywhere on an interval, then $f(x)$ is constant on that interval.

Corollary 2. If $f'(x) = g'(x)$ everywhere on an interval, then $f(x) = g(x) + \text{Constant}$.

Corollary 3. If $f'(x) > 0$ everywhere on an interval, then $f(x)$ is increasing on the interval.

If $f'(x) < 0$ everywhere on an interval, then $f(x)$ is decreasing on the interval.