MATH 1420: WORKSHEET FOR SECTION 3.6

More rules for derivatives: the chain rule

The chain rule, prime notation.

$$\frac{\mathrm{d}}{\mathrm{d}x} y(u(x)) = y'(u(x)) \cdot u'(x)$$

The chain rule, Leibniz notation.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

A special case: horizontal transformations.

$$\frac{\mathrm{d}}{\mathrm{d}x} y(ax+b) = ay'(ax+b), \qquad a, b \text{ are constants}$$

The chain rule is used to differentiate a function that is given by composition. For example, consider the function $g(x) = \sqrt{x^3 - x}$. You want to think of $g(x)$ as being given by composotion: $g(x) = y(u(x))$, where $y(u) = \sqrt{u}$ and $u(x) = x^3 - x$. So the derivative will be

$$g'(x) = \frac{3x^2 - 1}{2\sqrt{x^3 - x}},$$

where the numerator is $u'(x)$ and the denominator gives you $y'(u(x))$.

Here's some problems to practice the chain rule.

(1) Find the derivative of $f(x) = e^{\ln(b)x}$, where $b \neq 1$ is a positive constant. Also find the derivative of $g(x) = b^x$. Compare your answers.

(2) Differentiate $a(x) = \sqrt{x^3 + x + 1}$

(3) Find the derivative of $\sin(\frac{180}{\pi}x)$ and $\cos(\frac{180}{\pi}x)$. What does this tell you about the calculus of trig functions when you use degrees instead of radians?

(4) Differentiate $b(x) = \cos^3(2x^2)$. (Hint: you need to use the chain rule twice!)

(5) Differentiate $c(x) = \frac{1}{x^2+2x+1}$ by using the chain rule.

(6) Differentiate

$$N(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Then differentiate

$$N_{\mu,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma)}, \qquad \mu, \sigma \text{ are constants.}$$

(These functions are used in probability and statistics. $N_{\mu,\sigma}(x)$ is the *probability distribution function* for a *normal random variable* with mean μ and standard deviation σ.)

(7) The function $t(x) = \sqrt{r^2 - x^2}$ gives the top half of a circle of radius r centered at the origin. Write an equation for the tangent line to the circle of radius r at the point with x-coordinate $r/2$.