Math 1420: Study Guide for Final Exam

General comments:

- The format for the final exam is similar to the midterms, except it's a little longer. On the other hand, you have 120 minutes rather than 75 minutes.
- Calculators and other electronic devices are not allowed for the exam.
- You may bring a single 3 by 5 index card with formulas, notes, or whatever else you want on it. Write your name on it, and turn it in to me with your exam.
- The exam is about 1/4 to 1/3 new material, with the remainder cumulative over the material from the three midterms.
- Show your work! For one, understanding the process and how to communicate your logic to others is more important than being able to produce a correct answer with no explanation. For another, I cannot give partial credit if you show no work.

Here's what you should know for the new material since the last midterm.

1. Conceptual Understanding
    - The meaning of definite integral, and its connection to area.
2. Formal Understanding
    - How the fundamental theorem of calculus gives a connection between integrals and derivatives. How to use it to determine the derivative of a function defined by integration.
3. Rules for Calculations
    - How to use the substitution rule to compute indefinite and definite integrals.
4. Approximations and Applications
    - How to set up a Riemann sum. How to calculate a left or right Riemann sum (with small N, about $N = 4$).

For the cumulative material, you should know everything from the previous study guides. Let me highlight some especially important material.

1. Conceptual Understanding
    - The geometric meaning of the derivative, and its connection to slope.
    - How to determine limits given the graph of a function.
2. Formal Understanding
    - How to find maximums and minimums, as well as critical points. How to determine where a function is increasing, decreasing, concave up, concave down.
3. Rules for Calculations

- How to calculate antiderivatives of basic functions.
- How to calculate derivatives.
- L'Hôpital's rule for calculating limits with $0/0$ and ∞/∞ indeterminate form.

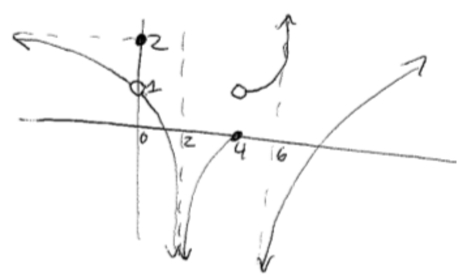
4. Approximations and Applications

   - How to solve optimization problems

Here are some sample questions similar to what you should expect to see on the exam.

1. Use the graph of the function $f(x)$ below to determine the limits

$$\begin{array}{ccc} \lim_{x\to-\infty} f(x), & \lim_{x\to 0} f(x), & \lim_{x\to 2} f(x), \\ \lim_{x\to 4^-} f(x), & \lim_{x\to 4^+} f(x), & \lim_{x\to 4} f(x), \\ \lim_{x\to 6^-} f(x), & \lim_{x\to 6^+} f(x), & \lim_{x\to 6} f(x). \end{array}$$



2. Consider the function

$$f(x) = \frac{2x(x+1)}{(x+1)(x-2)^2}.$$

   Calculate the limits

$$\lim_{x\to-\infty} f(x), \quad \lim_{x\to-1} f(x), \quad \lim_{x\to 0} f(x), \quad \lim_{x\to 2} f(x), \quad \lim_{x\to\infty} f(x)$$

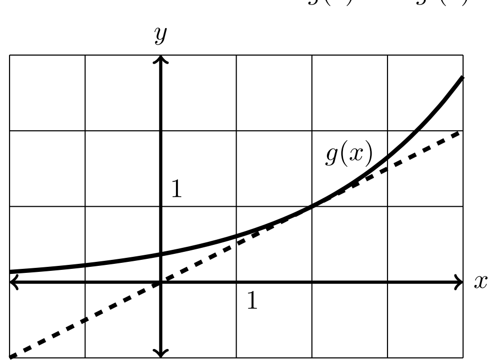
3. Calculate the limit

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x-1}-1}.$$

4. Use the squeeze theorem to compute the limit

$$\lim_{x\to 0} x^2 \sin(1/x).$$

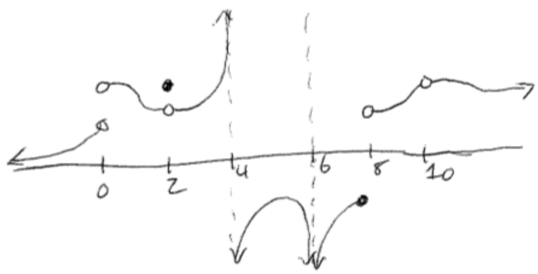
5. The function $g(x)$ is given by the following graph, with the tangent line at $x = 2$ drawn in. Use this information to determine $g(2)$ and $g'(2)$.



6. Calculate the integral $\int_{-2}^{2} \sqrt{4-x^2}\,\mathrm{d}x$. [Hint: draw a picture, use a geometry formula!]

7. Your car starts at rest and over the course of 2 minutes you accelerate to a speed of 60 miles per hour. Was there a time during those 2 minutes where your speed was exactly equal to your average speed over thoes 2 minutes? Justify your answer with a short explanation.

8. Use the graph below of the function $f(x)$ to list all the values of x where $f(x)$ has a discontinuity. For each, state what kind of discontinuity (removable, jump, or infinite) it is.



9. Consider the piecewise-defined function

$$f(x) = \begin{cases} x^2 + 4x & \text{if } x < 0 \\ 2^x - 1 & \text{if } 0 < x < 4 \\ 30 - x^2 & \text{if } 4 < x \end{cases}$$

This function is undefined and thus discontinuous at $x = 0$ and $x = 4$. For each discontinuity, determine whether it is a removable discontinuity or a jump discontinuity. Justify your answers with limit calculations.

10. You know that $f(x)$ is continuous on the interval $[-3, 3]$, that $f(-3) = -10$, and $f(3) = 1$. Can you conclude that $f(x)$ has a zero? Justify your answer with a sentence.

11. The function $F(x)$ is defined as $F(x) = \displaystyle\int_x^{420} \cos(e^t)\,\mathrm{d}t$. Determine $F'(x)$.

12. The function $G(x)$ is defined as $G(x) = \displaystyle\int_x^{e^x} \ln(t)\,\mathrm{d}t$. Determine $G'(x)$.

13. Consider the function $h(x) = x^3 - 3x^2 + 36x - 100$. Find the location of all local maximums and minimums of $h(x)$.

14. For this same function $h(x)$, where is it increasing? Where is it decreasing? Give your answer in interval notation.

15. For this same $h(x)$, determine where it is concave up, and where it is concave down. Where are its inflection points?

16. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = x^3 + 3$.

17. Use implicit differentiation to find the derivative of $q(x) = (\sqrt{x})^x$.

18. Calculate the indefinite integral

$$\int 2xe^{\sin x + x^2} + \cos x\, e^{\sin x + x^2}\,\mathrm{d}x.$$

19. Calculate the definite integral

$$\int_0^2 \frac{2x}{x^2+1}\,\mathrm{d}x.$$

20. Calculate the indefinite integral

$$\int e^t - 4t^2 + \cos t \, \mathrm{d}t.$$

21. Calculate the definite integral

$$\int_{-1}^{1} 4x^3 - 6x + 1 \, \mathrm{d}x.$$

22. Calculate the derivative of

$$a(x) = x^{40} - 10x^{11} + x^2 + 130.$$

23. Calculate the derivative of

$$b(x) = e^{x^2}.$$

24. Calculate the derivative of

$$c(x) = \frac{\ln x}{x^2}$$

25. Calculate the derivative of

$$d(x) = \tan(4x) - \arctan(2x).$$

26. Use L'Hôpital's rule to compute the limit

$$\lim_{x \to 0} \frac{\cos x - 1}{x}.$$

27. Use L'Hôpital's rule to compute the limit

$$\lim_{x \to \infty} \frac{e^{x/100}}{50x^2}.$$

28. Consider the function $q(x) = x^2 + 4x$. Write the right Riemann sum to approximate the area under this curve on the interval $[0, 100]$, with $N = 1000$ regions. Substitute in the definitions of the various pieces so that your summand depends only on the index variable i for the sum.

29. For the same $q(x)$, compute the left Riemann sum to approximate the integral

$$\int_0^6 q(x) \, \mathrm{d}x,$$

using $N = 3$ regions.

30. Find the point on the line $y = x + 1$ which minimizes the distance to the point $(1, 4)$.

31. Find the two numbers x, y both between 0 and 10 which minimizes the value $x^3 + y^2$ given the constraint $x + y = 10$.

32. A perfectly spherical balloon is being inflated at a rate of 2 m^3 per second. What is the rate of change of the radius of the balloon at the moment when the radius is 10 m?

33. Find the slope of the curve $e^{xy} + xy + x = 1$ at the point $(0, 2)$.