SAMPLE WRITTEN HOMEWORK

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This is a sample document to show how your written homework should look. Remember that the purpose of these is to get practice with communicating mathematics. The two things you should focus on are (1) making sure your mathematics is correct and (2) making sure your writing is clear and you fully explain things.

I did this as a typed-up document for readability, but I'm not asking you to submit typewritten work. (It can be a bit of a learning curve to learn tools for typesetting mathematical formulas, and I'm not asking you to do this.) Handwritten work is okay. I'm not picky about formatting details. So long as your name is on the paper and your work is readable, it's all good.

Specific comments below are in footnotes.

Problem. *Consider the function*

$$f(x) = \begin{cases} x^3 & \text{if } x < 1 \\ \cos(x-1) & \text{if } x > 1. \end{cases}$$

This function is not defined at $x = 1$. Is there a way to define a value for $f(1)$ which makes the function continuous at all real numbers?[1]

Solution. Yes this can be done, by setting $f(1) = 1$.[2]

On the interval $(-\infty, 1)$ the function $f(x)$ is identical to x^3. Because x^3 is a polynomial it is continuous everywhere, which means that $f(x)$ is continuous on $(-\infty, 1)$.[3] Similarly, on the interval $(1, \infty)$ the function $f(x)$ is identical to $\cos(x-1)$. This is a composition of functions which are continuous everywhere, so it is also continuous everywhere.[4] Thus, $f(x)$ is continuous on $(1, \infty)$. We[5] have seen that $f(x)$ is continuous on $x < 1$ and $x > 1$, so the only remaining work is to see how to make the function continuous at $x = 1$.

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[1] Convention in mathematics is to tell the reader what you are doing before you do it. Your write-up should start with a clear statement of the problem you are solving.

[2] In line with the previous comment, if the problem asks you a question it's good to state the answer before you explain why it's the correct answer.

[3] A lot of mathematical writing comes down to sentences of the form "X because Y." What you should keep in mind is, you don't just want to say what's true, you want to explain *why* it is true.

[4] The 'why' in this sentence is a fact we learned in class. When quoting facts from class you don't need to explain why they are true.

Maybe you remember something is true but you don't remember exactly why. In that case, it's a good idea to look at your notes from class or at the textbook to remind yourself of the explanation so you can provide it

[5] Some people like to write mathematical explanations with plural pronouns—we meaning the author and the reader together. If you prefer to write I/me, that's also fine. There's multiple correct ways to write.

In order for $f(x)$ to be continuous at $x = 1$ we need that the two one-sided limits are the same. In symbols,[6]

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x).$$

These limits are easy to calculate, again using the piecewise definition of $f(x)$. Namely,[7]

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^3 = 1^3 = 1$$

and

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \cos(x-1) = \cos(1-1) = 1.$$

For the first calculation we used[8] that x^3 is continuous everywhere to know that the limit as $x \to 1^-$ is just 1^3, and similarly for the other direction's calculation.[9] Because both one-sided limits are equal to 1, if we set $f(1) = 1$ we get a function which is continuous at all real numbers. □

Having seen a good solution let me contrast with a bad solution.

Bad Solution. Yes, set $f(1) = 1$.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^3 = 1^3 = 1$$
$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \cos(x-1) = \cos(1-1) = 1$$ □

This has the correct answer and it shows the calculations that are the key step to getting that answer. But it doesn't explain why these are the right calculations to do. Imagine yourself as the audience. If you didn't already understand why those are the right calculations to do this wouldn't explain that to you.

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[6]Earlier the math symbols "in-line", like they were just ordinary words or phrases in a paragraph. Now I'm putting some on their own line. This is good to do for larger mathematical expressions that would otherwise be hard to read.

[7]Basic computational and arithmetic facts don't need to be explained. For example, it's fine to just write $\cos(1-1) = 1$ without explaining why $\cos(0)$ is equal to 1. Think: you're explaining your solution to a classmate in this class. You should explain the new ideas from calculus, but you don't need to explain ideas from trigonometry or algebra they already know from previous classes.

I put these step-by-step calculations each on one line. But you may prefer to put them across multiple lines, especially if there's more steps.

[8]Even though the calculations were all done on one line it's still good to explain the big idea for what makes the calculations work.

[9]When you use math symbols in a sentence the whole thing should sound like complete English sentences/phrases when read out loud. For example, the phrase earlier in the sentence would read "...the limit as x approaches one from the right is just one cubed..." If you read a sentence with math symbols out loud and it sounds ungrammatical, you should rewrite it!