

Math 1410: Study Guide for Midterm 3

November 7, 2022

General comments:

- It's important to distinguish between the different types of objects we're dealing with—numbers, points, functions, and intervals—and understand what's being asked about. For example, if a problem asks you to find the y -intercept of a function, that is a *point*, while a problem asking you to find the initial value of a function was a *number*.
- Questions about domains, ranges, where a function is increasing, etc. are asking for intervals, and you should write your answers in interval notation.
- The questions are written with the fact that calculators are not allowed in mind. In particular, questions involving inverse trig will use special angles you should know the values for.
- Show your work! I cannot give partial credit if you don't.
- For a lot of these problems, understanding the unit circle picture will be very helpful. Remember: sin is about the y -coordinate, cos is about the x -coordinate, and tan is about the slope.

The problems on the exam are broken up by learning objective. Here's what you should know for each objective.

1. Rates of change.

- Understand how the trig functions change across a quadrant.
- How to compute average rates of change for trig functions with numbers given for the endpoints.
- *You will not be asked to compute the ARC across a generic interval $[x, x + h]$.*

2. Pointwise behavior of functions.

- How to find initial values/ y -intercepts of trig functions.
- How to find zeros/ x -intercepts of trig functions.
- How to solve forward and inverse problems involving trig functions and inverse trig functions.

3. Interval behavior of functions.

- Understand where trig functions are increasing, decreasing, concave up, and concave down.
- Understand the domain and range of trig functions.
- Understand extreme points (maximums and minimums) of trig functions.
- Understand how vertical asymptotes behave for trig functions.

4. Graphing.

- Given an equation for a trig function how to find its amplitude, period, and vertical/horizontal shifts.
- How to sketch a graph of a trig function.

- Given a graph of a trig function, determine an equation for the graph.
- *With regard to graphing, the most important trig functions are sine and cosine. So I would suggest those be the ones you spend the most time studying.*

5. Rewriting equations of functions.

- How to simplify expressions involving multiple trig functions.
- How to use identities to rewrite expressions involving trig functions.
- *I will not give you a formula sheet with identities. You should know the reciprocal and ratio identities (= the definitions of the trig functions) and the Pythagorean identity.*

6. Transcendental operations.

- Understand standard position for angles, how to find coterminal angles, and how to find reference angles.
- How to compute trig functions of angles using the unit circle definitions and the XYR definitions.
- How to compute trig functions of an angle knowing about other trig functions of that angle.
- How to compute exact values trig functions of quadrantal angles or angles with reference angle $\pi/6$, $\pi/4$, or $\pi/3$.
- Understand what quadrants are the outputs for different inverse trig functions.
- *This is mostly the material we covered in the first week of this unit. In particular, for some problems you will need to know sin and cos for the special angles $\pi/6$, $\pi/4$, and $\pi/3$. See below for a reminder.*

7. Function algebra.

- *This objective will not be part of this exam.*

8. Inequalities and functions.

- *This objective will not be part of this exam.*

For some problems, you will need to know the exact values of trig functions for special angles. Here's some tips on how to remember these.

- You can compute what happens with the other trig functions if you know what happens with sin and cos. So you only need to remember those two functions' values.
- For quadrantal angles, the x - and y -coordinates are either 0 or ± 1 , depending on what direction the angle points.
- The magnitude of the output of a trig function is the same as what happens at its reference angle, and the quadrant of the angle determines its sign (positive versus negative). So you only need to memorize the values for angles in quadrant 1.
- In short, these are the only six values to memorize:

	$\pi/6$	$\pi/4$	$\pi/3$
$\sin(t)$	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
$\cos(t)$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$

You can remember these by the 1, 2, 3 pattern (3, 2, 1 for cosine). As a last resort in case you suddenly blank mid-exam, you can rederive these using geometry. The $\pi/4$ angle comes from an isosceles right triangle where the two legs are the same length, and the $\pi/6$ and $\pi/3$ angles come from half an equilateral triangle.

- Let me also say, if you do forget a value, that's not a huge deal. For example, if the question is to solve $\cos x = \sqrt{2}/2$ and you can't remember that $\cos(\pi/4) = \sqrt{2}/2$, just write $\arccos(\sqrt{2}/2)$ and you can still get most the points (assuming you do the other work correctly).