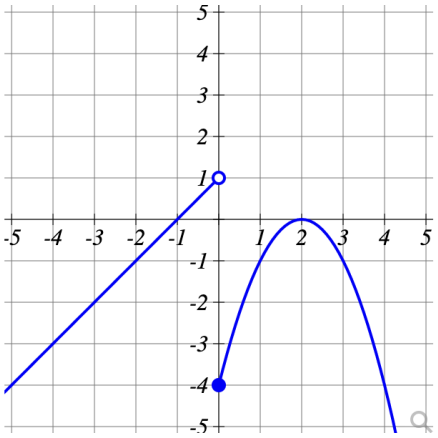
Functions as Quantities Changing with Each Other

1. The function $f(x)$ is graphed below:



(a) Draw a line that is secant to f at $x = 1$ and $x = 4$.

(b) What is the average rate of change of f between $x = 1$ and $x = 4$.

(c) Describe how f is changing over $(-\infty, 0)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

(d) Describe how f is changing over $(0, 2)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

2. The continuous function $m(t)$ is represented in the table below:

t	0	1	2	3	4	5	6	7
$m(t)$	11	9	7	5	6	9	14	24

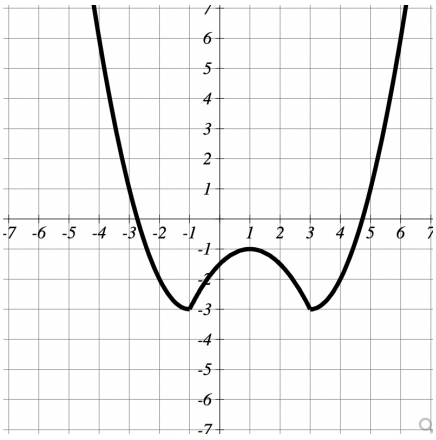
(a) What is the average rate of change of m over $[2, 5]$?

(b) Describe how $m(t)$ is changing over $(0, 3)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

(c) Describe how $m(t)$ is changing over $(4, 7)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

3. Let $f(x) = x^2 + 9$. Find the average rate of change of f over $[x, x + h]$. You must show a step-by-step solution to receive credit.

4. The function $f(x)$ is graphed below:



(a) Describe how f is changing over $(-\infty, -1)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

(b) Describe how f is changing over $(-1, 1)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

(c) Describe how f is changing over $(1, 3)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

(d) Describe how f is changing over $(3, \infty)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

(e) The rate of change at $x = -3$ is...
- ◯ positive
- ◯ negative
- ◯ zero

(f) The rate of change at $x = 1$ is...
- ◯ positive
- ◯ negative
- ◯ zero

5. The continuous function $m(t)$ is represented in the table below:

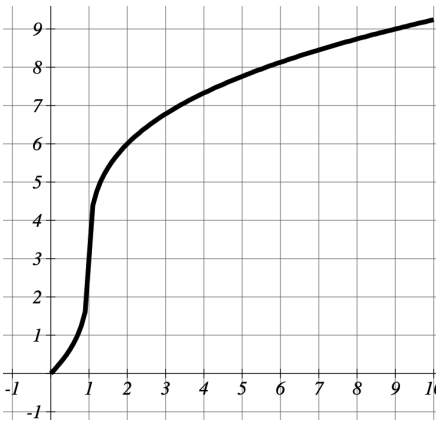
t	0	1	2	3	4	5	6	7
$m(t)$	3	-1	-5	-9	-5	-2	1	2

(a) What is the average rate of change of m over $[2, 5]$?

(b) Describe how $m(t)$ is changing over $(0, 3)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

(c) Describe how $m(t)$ is changing over $(3, 7)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

6. A random kid from Peoria posted a video on social media that went viral. The graph below shows his number of followers over time. For the function graphed x is the number of days since he posted the video and y is his number of followers, in millions.

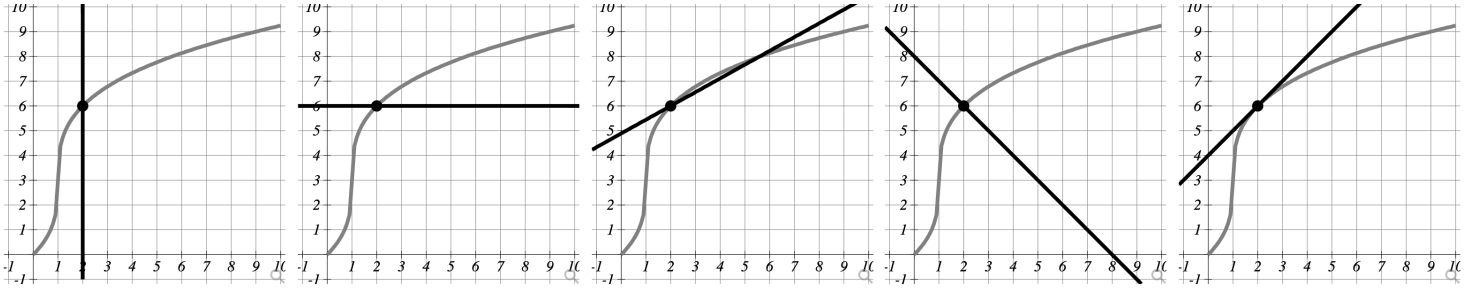


(a) How many followers does he have two days after posting the video?

(b) Go back to the graph above part A and draw a line that is secant to the curve at $x = 2$ and $x = 9$.

(c) On average how many followers did he gain each day between during the time period that started 48 hours after he went viral and ended 216 hours after he went viral?

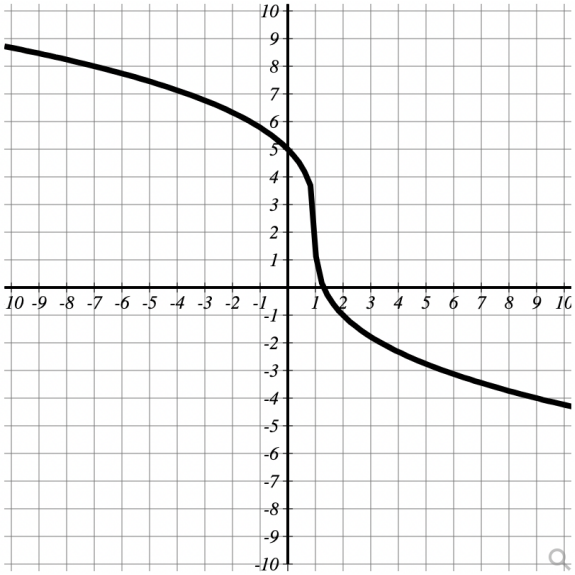
(d) Examine the graphs below and determine which one shows the line that is tangent to the graph at $x = 2$. Draw a box around the correct graph.



(e) At what rate, in followers gained per day, was he gaining followers exactly 48 hours after posting his video?

7. Let $g(x) = 3x^2 - 5x + 4$. Find the average rate of change of g over $[x, x+h]$. You must show a step-by-step solution to receive credit.

8. The function $f(x)$ is graphed below:



(a) What is the average rate of change of f over $[-7, 0]$?

(b) The slope of the line tangent to the graph of f at $x = 1.5$ is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

(c) The slope of the line tangent to the graph of f at $x = 5$ is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

(d) If you compare the line tangent to the graph of f at $x = 1.5$ and the line tangent to the graph of f at $x = 5$, which has a greater slope?
- ◯ The line tangent to f at $x = 1.5$ has a greater slope.
- ◯ The line tangent to f at $x = 5$ has a greater slope.
- ◯ The lines tangent to the graph of f at $x = 1.5$ and $x = 5$ have slopes that are equal.
- ◯ The graph doesn't give us enough information.

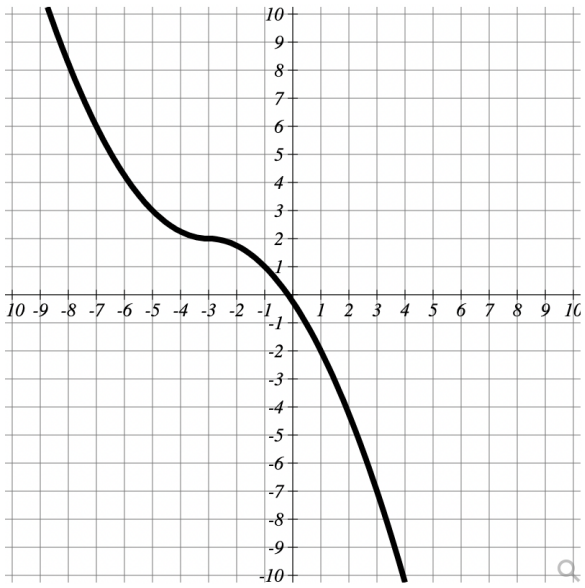
(e) Describe how f is changing over $(-\infty, 1)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

(f) Describe how f is changing over $(1, \infty)$.
- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

9. Let $f(x) = 2x^3$. Find the average rate of change of f over $[x, x + h]$. You must show a step-by-step solution to receive credit.

10. Select **ONE** of the two following problems to work. You must show a step-by-step solution to receive credit. You must circle the letter of the one that you are working.
If you attempt both and expect your instructor to grade the better one, you will receive **NO CREDIT**.
If you attempt both, you **MUST** circle the letter of the one that you want your instructor to grade.
    (a) Let $f(x) = \sqrt{x}$. Find the average rate of change of f over $[x, x+h]$.
    (b) Let $f(x) = \frac{1}{x}$. Find the average rate of change of f over $[x, x+h]$.

11. The function $f(x)$ is graphed below:

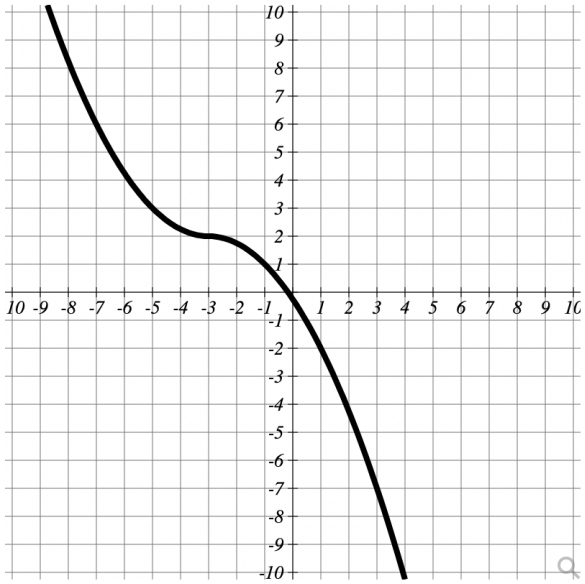


    (a) What is the average rate of change over $[-7, -5]$?

    (b) What is the average rate of change over $[-5, -3]$?

    (c) Is the average rate of change greater over $[-7, -5]$ or $[-5, -3]$?
    - ◯ The rate of change is greater over $[-7, -5]$.
    - ◯ The rate of change is greater over $[-5, -3]$.
    - ◯ The rate of change over $[-7, -5]$ and $[-5, -3]$ are equal.
    - ◯ The graph doesn't give us enough information.

    (d) Describe how f is changing over $(-\infty, -3)$.
    - ◯ increasing at a constant rate
    - ◯ increasing at an increasing rate
    - ◯ increasing at a decreasing rate
    - ◯ decreasing at a constant rate
    - ◯ decreasing at an increasing rate
    - ◯ decreasing at a decreasing rate



(a) The instantaneous rate of change of f at $x = -2$ is...

- ◯ Positive
- ◯ Negative
- ◯ Zero

(b) The instantaneous rate of change of f at $x = 2$ is...

- ◯ Positive
- ◯ Negative
- ◯ Zero

(c) Is the instantaneous rate of change greater at $x = -2$ or $x = 2$?

- ◯ The rate of change is greater at $x = -2$.
- ◯ The rate of change is greater at $x = 2$.
- ◯ The rate of change is greater at $x = -2$ and $x = 2$ are equal.
- ◯ The graph doesn't give us enough information.

(d) Describe how f is changing over $(-3, \infty)$.

- ◯ increasing at a constant rate
- ◯ increasing at an increasing rate
- ◯ increasing at a decreasing rate
- ◯ decreasing at a constant rate
- ◯ decreasing at an increasing rate
- ◯ decreasing at a decreasing rate

12. Let $f(x) = 2x^4 + 1$. Find the average rate of change of f over $[x, x + h]$. You must show a step-by-step solution to receive credit and fully simplify your answer.

13. Select **ONE** of the two following problems to work. You must show a step-by-step solution to receive credit. You must fully simplify your answer. You must circle the letter of the one that you are working. If you attempt both and expect your instructor to grade the better one, you will receive **NO CREDIT**. If you attempt both, you **MUST** circle the letter of the one that you want your instructor to grade.
    (a) Let $f(x) = \sqrt{x}$. Find the average rate of change of f over $[x, x + h]$.
    (b) Let $f(x) = \frac{1}{x}$. Find the average rate of change of f over $[x, x + h]$.

14. Find the average rate of change of $g(\theta) = \cos\theta$ between $\theta = 0$ and $\theta = \pi$.

15. The angle θ is sweeping through Quadrant III in the **negative** direction. Describe each of the following:

(a) $\sin\theta$ is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

(b) $\sin\theta$ is...
- ◯ Increasing
- ◯ Decreasing
- ◯ Constant

(c) The rate at which $\sin\theta$ is changing is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

(d) $\cos\theta$ is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

(e) $\cos\theta$ is...
- ◯ Increasing
- ◯ Decreasing
- ◯ Constant

(f) The rate at which $\cos\theta$ is changing is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

16. Find the average rate of change of $g(\theta) = \cos\theta$ between $\theta = 0$ and $\theta = \pi$.

17. The angle θ is sweeping through Quadrant III in the **negative** direction. Describe each of the following:

(a) $\sin\theta$ is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

(b) $\sin\theta$ is...
- ◯ Increasing
- ◯ Decreasing
- ◯ Constant

(c) The rate at which $\sin\theta$ is changing is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

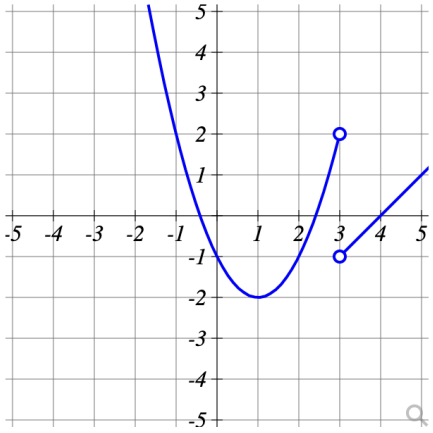
(d) $\cos\theta$ is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

(e) $\cos\theta$ is...
- ◯ Increasing
- ◯ Decreasing
- ◯ Constant

(f) The rate at which $\cos\theta$ is changing is...
- ◯ Positive
- ◯ Negative
- ◯ Zero

Behavior of Function at a Point

18. The graph of k is given below:



Find the requested values. If any of them are not integer values, you may give approximate decimal answers.

(a) The zeros of k

(b) The value of $k(3)$

(c) The initial value of k

(d) The value of c if $k(c) = -1$

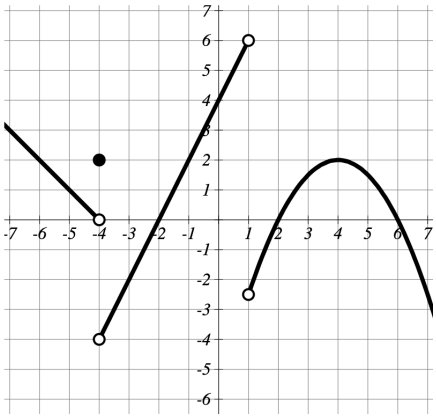
19. Let $w(x) = a - 2(x + 5b)$ for constants $a, b \in \mathbb{R}$. Find each of the following:

(a) The y-intercept(s) of w.

(b) The zero(s) of w.

20. Find the point(s) on the graph of f with a y-value of 5 if $f(x) = x^2 - 4x + 6$.

21. The function of $k(t)$ is given below:



Find each of the following:

(a) The zero(s) of k

(b) $k(1)$

(c) c when $k(c) = 2$

(d) The point(s) on the graph of k where $t = -2$

(e) $k(-1)$

(f) The initial value(s) of k

(g) The t-intercept(s)

22. The function g is a continuous function that maps values of $x > -2$ to values of $y \leq 10$. A table is given below for the function g:

x	-1	0	1	2	3	4	5	6
$g(x)$	3	2	0	-4	-5	-2	0	-2

Find the following:

(a) $g(6)$

(b) The initial value of g

(c) The x-intercepts of g

(d) The points on the graph of g where $y = -2$

(e) $g(-5)$

(f) Zero(s) of g

(g) Points on the graph of g where $x = 2$

23. (30 points) Let $h(x) = (x + n)^2 + m$ where $n, m \in \mathbb{R}$ and $m < 0$. Find each of the following:

(a) The y-intercept(s) of h

(b) The x-intercept(s) of h

24. Let $s(t) = t^2 + 2t + 4$.

(a) Find the value of a such that $s(a) = 5$.

(b) Find the vertex of s.

25. The function f is $f(x) = 5(x + 3)(x - 4)^2(2x - 1)$ when written in factored form and $f(x) = 10x^4 - 55x^3 - 55x^2 + 520x - 240$ when f is written in standard descending order. Find each of the following:

(a) The y-intercept

(b) The zeros and their multiplicities

26. The point(s) on the graph of $f(x) = \dfrac{2x + 3}{x + 1}$ where y is $\frac{8}{3}$.

27. Let $g(x) = \dfrac{(x + 3)^2(x - 1)}{5x(2x - 3)(x - 1)}$. Find each of the following or state that none exist.

(a) The horizontal asymptote

(b) The vertical asymptote(s)

(c) The x-intercept(s)

(d) Any holes in the graph of g.

28. Let $f(x) = \sqrt{3} + 2\cos x$

(a) Find the y-intercept of f.

(b) Find the all zeros of f in $(-\infty, \infty)$.

29. The function f is $f(x) = 10(x + 8)(x + 3)^2(2x - 5)^3(x - 1)$ when written in factored form and $f(x) = 80x^7 + 440x^6 - 2860x^5 - 6350x^4 + 33490x^3 + 11950x^2 - 126750x + 90000$ when f is written in standard descending order. Find each of the following:

(a) The x-intercepts and their multiplicities

(b) The initial value

30. Let $g(t) = 5t^4 + 8$.

(a) Find the point of the graph of g where $t = 2a$ for $a \in \mathbb{R}$.

(b) Find the point of the graph of g where $y = c + 8$ for $c \in \mathbb{R}$.

31. Let $g(x) = \dfrac{x(2x+9)(x+1)}{(x-2)^2(x+3)(x+1)}$. Find each of the following or state that none exist.
    (a) The horizontal asymptote
    (b) The vertical asymptote(s)
    (c) The x-intercept(s)
    (d) Any holes in the graph of g.

32. Find the value of θ in each of the following situations.
    (a) $\theta = \arcsin(-\dfrac{1}{2})$
    (b) $\sin\theta = -\dfrac{1}{2}$ where θ is in $[0, 2\pi]$
    (c) $\sin\theta = -\dfrac{1}{2}$ where θ is in $(-\infty, \infty)$

33. Let $f(x) = 5 - 10\log_9(\frac{x-1}{5})$.
    (a) Find the zero(s) of f.
    (b) Find the y-intercept of f.

34. Find the point where the functions $f(x) = \ln(2x+5)$ and $g(x) = \ln(4x+13)$ intersect.

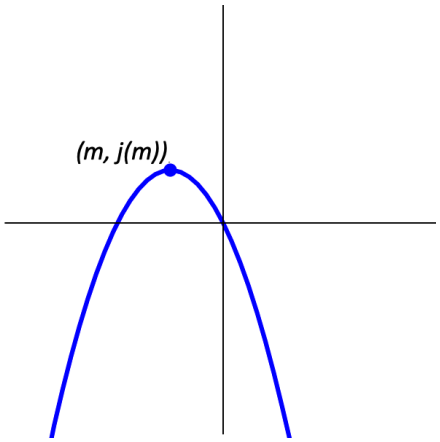
35. Find the initial value of $k(a) = \dfrac{2e^{a-3}}{5}$.

36. Find the t-intercept of $h(t) = -2 + 7^{t+8}$.

37. Find the point on the graph of g where $g(x) = -21$ when $g(x) = 3(6)^{2x-9}$.

Behavior of Function Over an Interval

38. The function $j(x)$ is graphed below:



Find each of the items to the right for this function.

(a) The increasing intervals of j.
(b) The decreasing intervals of j.
(c) What is the domain of j?
(d) What is the range of j?
(e) Complete the following sentence:
The function j has an absolute *minimum/maximum* (circle one) of _____ at $x =$ _____.

39. Let $g(x) = 2(x-4)^2 - 8$. State each of the following:

(a) The increasing intervals of g.
(b) The decreasing intervals of g.
(c) Is g concave up or concave down?
◯ concave up
◯ concave down
◯ neither
(d) What is the domain of g?
(e) What is the range of g?
(f) Fill in the blanks below:
$x \to -\infty, g(x) \to$ _____
$x \to \infty, g(x) \to$ _____
(g) Complete the following sentence:
The function g has an absolute *minimum/maximum* (circle one) of _____ at $x =$ _____.

40. Let $f(x) = -x^2 + 4x - 5$. State each of the following. *Hint: Finding the vertex may help you answer the following questions.*

(a) The increasing intervals of f.
(b) The decreasing intervals of f.
(c) Is f concave up or concave down?
◯ concave up
◯ concave down
◯ neither
(d) What is the domain of f?
(e) What is the range of f?
(f) Fill in the blanks below:
$x \to -\infty, f(x) \to$ _____
$x \to \infty, f(x) \to$ _____
(g) Complete the following sentence:
The function f has an absolute *minimum/maximum* (circle one) of _____ at $x =$ _____.

41. f is a quadratic function with a vertex of $(-2, 4)$, zeros of -6 and 2, and an initial value of 3. Find each of the following:

(a) The increasing intervals of f.

(b) The decreasing intervals of f.

(c) Is f concave up or concave down?

- ◯ concave up
- ◯ concave down
- ◯ neither

(d) What is the domain of f?

(e) What is the range of f?

(f) Fill in the blanks below:
$x \to -\infty, f(x) \to$ _____
$x \to \infty, f(x) \to$ _____

(g) Complete the following sentence:
The function f has an absolute *minimum/maximum* (circle one) of _____ at $x =$ _____.

42. $g(x) = a + \frac{1}{b}(x+c)^2$ where $a, b, c \in \mathbb{R}$ and $a \leq 0$, $b > 0$, and $c \neq 0$.

(a) The increasing intervals of g.

(b) The decreasing intervals of g.

(c) Is g concave up or concave down?

- ◯ concave up
- ◯ concave down
- ◯ neither

(d) What is the domain of g?

(e) What is the range of g?

(f) Fill in the blanks below:
$x \to -\infty, g(x) \to$ _____
$x \to \infty, g(x) \to$ _____

(g) Complete the following sentence:
The function g has an absolute *minimum/maximum* (circle one) of _____ at $x =$ _____.

43. Let $h(x) = -x^2 + 4x - 1$.

(a) The increasing intervals of h.

(b) The decreasing intervals of h.

(c) Is h concave up or concave down?

- ◯ concave up
- ◯ concave down
- ◯ neither

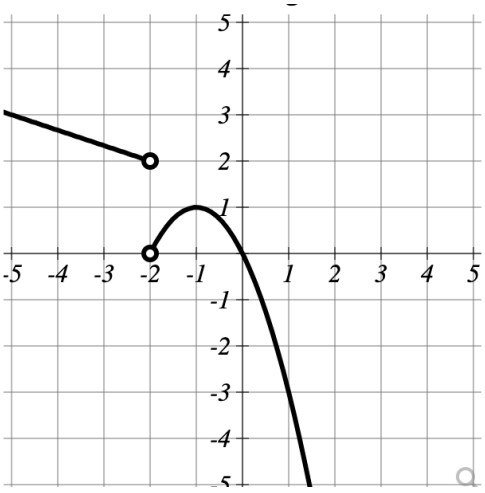
(d) What is the domain of h?

(e) What is the range of h?

(f) Fill in the blanks below:
$x \to -\infty, h(x) \to$ _____
$x \to \infty, h(x) \to$ _____

(g) Complete the following sentence:
The function h has an absolute *minimum/maximum* (circle one) of _____ at $x =$ _____.

44. The graph $m(t)$ is graphed below:

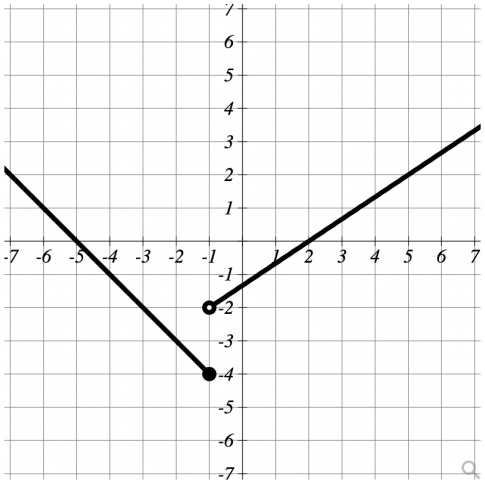


State the domain and range of m.

(a) Domain

(b) Range

45. Let $q(x)$ be defined by the graph below:



(a) The increasing intervals of q.

(b) The decreasing intervals of q.

(c) What is the domain of q?

(d) What is the range of q?

(e) Fill in the blanks below:
$x \to -\infty, q(x) \to$ _____
$x \to \infty, q(x) \to$ _____

(f) Complete the following sentence:
The function q has an absolute *minimum/maximum* (circle one) of _____ at $x =$ _____.

46. Let $g(n) = \dfrac{1}{n-5} + 2$. Find each of the following. Hint: It may be helpful to envision the function's graph or create a quick sketch of it.

(a) $\lim_{n \to -\infty} g(n) =$

(b) $\lim_{n \to 5^-} g(n) =$

(c) $\lim_{n \to 5^+} g(n) =$

(d) $\lim_{n \to 5} g(n) =$

(e) $\lim_{n \to \infty} g(n) =$

(f) Over the interval $(-\infty, 5)$ g is...
- ◯ concave up
- ◯ concave down
- ◯ neither

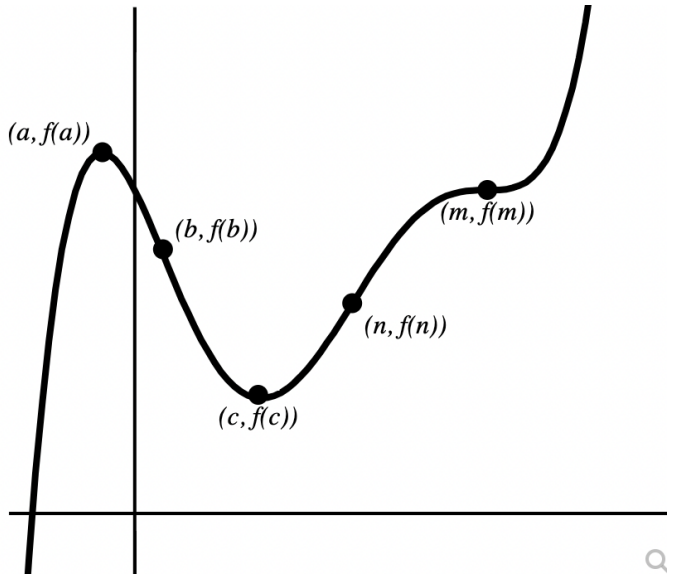
(g) Over the interval $(5, \infty)$ g is...
- ◯ concave up
- ◯ concave down
- ◯ neither

47. Let $k(t) = \dfrac{t+3}{(t+3)(t-10)}$. Find each of the following.
    (a) The domain of k
    (b) The hole in the graph of k.
    (c) $k(-3) =$
    (d) $\lim_{t \to -3} k(t) =$

48. Let $f(x) = (x+5)^4 + 6$. State each of the following.
    (a) The domain of f.
    (b) The range of f.
    (c) Complete the following sentence:
    The function f has an absolute *minimum/maximum* (circle one) of _____ at $x =$ _____.

49. Let $p(x) = -\sqrt{x+5} - 2$.
    (a) The domain of p.
    (b) As $x \to \infty, p(x) \to$ _____
    (c) $p(x)$ is
        ◯ increasing
        ◯ decreasing
        ◯ constant
    (d) $p(x)$ is
        ◯ concave up
        ◯ concave down
        ◯ neither

50. The function f is graphed below:



Find each of the following. Write all intervals in interval notation.
    (a) Where is f increasing?
    (b) Where is f decreasing?

(c) Where is f concave up?

(d) Where is f concave down?

(e) Complete the following sentences:

The function f has a relative (also called a local) minimum of _____ at $x =$ _____.

The function f has a relative (also called a local) maximum of _____ at $x =$ _____.

51. Let $f(x) = \dfrac{a}{(x-c)^2}$ where $a < 0$ and $c > 0$. Find each of the following. Hint: It may be helpful to envision the function's graph or create a quick sketch of it.

(a) $\lim\limits_{x \to -\infty} f(x) =$

(b) $\lim\limits_{x \to c^-} f(x) =$

(c) $\lim\limits_{x \to c^+} f(x) =$

(d) $\lim\limits_{x \to c} f(x) =$

(e) $\lim\limits_{x \to \infty} f(x) =$

52. Let $m(a) = \dfrac{3}{a-10}, a \neq -5$. Find each of the following.

(a) The hole in the graph of m.

(b) The domain of m

(c) $m(10) =$

(d) $m(-5) =$

(e) $\lim\limits_{a \to 10} m(a) =$

(f) $\lim\limits_{a \to -5} m(a) =$

53. Describe the end behavior of each of the following functions.

(a) $j(x) = \dfrac{2x+5}{3x+8}$

as $x \to -\infty$, $j(x) \to$ _____

as $x \to \infty$, $j(x) \to$ _____

(b) $g(x) = -x^2(x+3)(x-8)^2$

as $x \to -\infty$, $g(x) \to$ _____

as $x \to \infty$, $g(x) \to$ _____

54. Find the domain of $h(n) = 5 - \frac{3}{4}\sqrt{10-5n}$.

55. Let $g(x) = \sin x, 0 \leq x \leq 2\pi$. Answer the following questions about the function g. Drawing a quick sketch of g may help you answer these questions.

(a) What is the range of g?

(b) Over what interval is g increasing?

(c) Over what interval is g decreasing?

(d) Over what interval is g concave up?

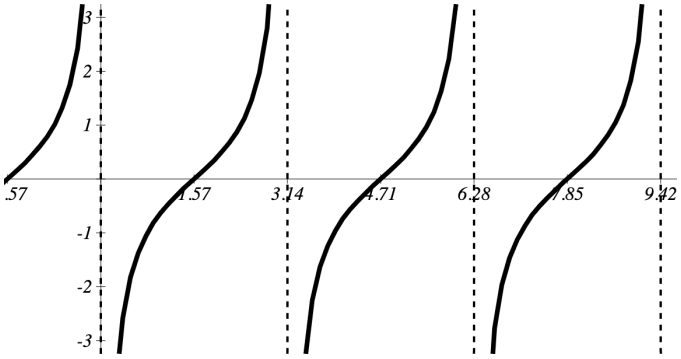
(e) Over what interval is g concave down?

(f) Describe the extrema of g

The function g has an absolute maximum of _____ *at* $x =$_____.

The function g has an absolute minimum of _____ *at* $x =$_____.

56. The graph $m(t)$ is graphed below:



Find each of the following.

(a) $\lim_{t\to\pi^-} m(t) =$

(b) $\lim_{t\to\pi^+} m(t) =$

(c) $\lim_{t\to\pi} m(t) =$

(d) What is the range of m?

57. Let $g(x) = \csc x, 0 \leq x \leq 2\pi$. Answer the following questions about the function g. Drawing a quick sketch of g may help you answer these questions.

(a) What is the range of g?

(b) Over what interval is g increasing?

(c) Over what interval is g decreasing?

(d) Over what interval is g concave up?

(e) Over what interval is g concave down?

(f) Describe the extrema of g

The function g has a relative maximum of _____ *at* $x =$_____.

The function g has a relative minimum of _____ *at* $x =$_____.

(g) $\lim_{x\to\pi^-} g(x) =$

(h) $\lim_{x\to\pi^+} g(x) =$

(i) $\lim_{x\to\pi} g(x) =$

(j) What is the range of g?

58. Let $k(t) = log_5(t + 4)$

(a) Is k increasing or decreasing?

(b) Is k concave up or concave down?

(c) What is the domain of k?

59. Let $f(x) = -3 - e^{-x}$. Find the following:

(a) $x \to -\infty, f(x) \to$ _____

(b) $x \to \infty, f(x) \to$ _____

60. Let $g(x) = ln(x)$. Find the following:

(a) $x \to -\infty, g(x) \to$ _____

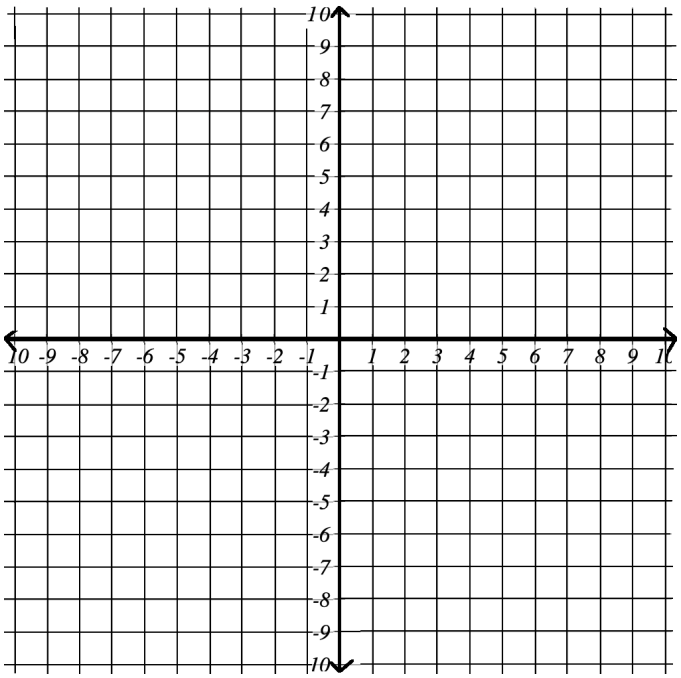
(b) $x \to \infty, g(x) \to$ _____

(c) $x \to 0^+, g(x) \to$ _____

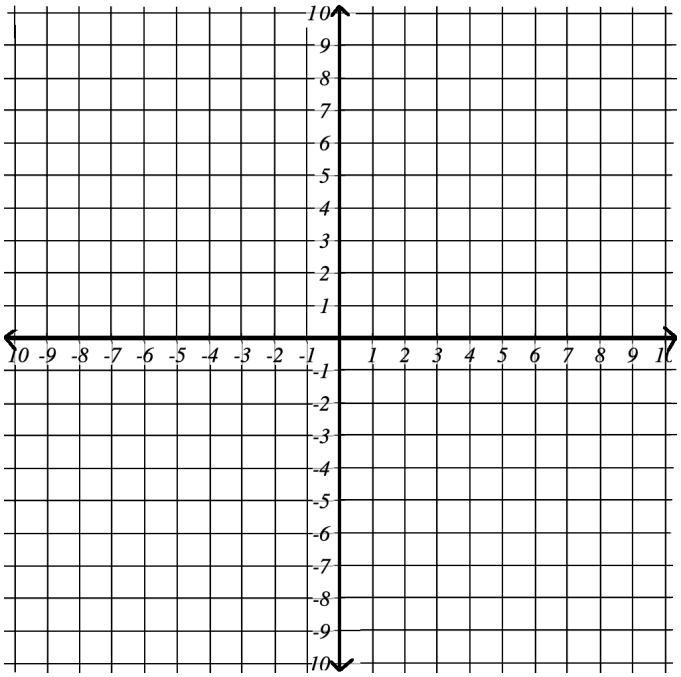
(d) $x \to 0, g(x) \to$ _____

Graphs of Functions

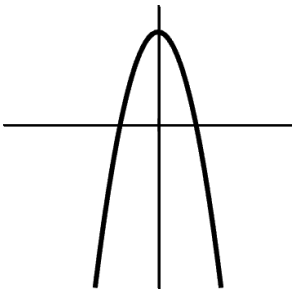
61. Graph the function $k(t) = \begin{cases} -5 - t & t < -3 \\ 5 & -3 \leq t < 0 \\ 2 & t > 0 \end{cases}$.



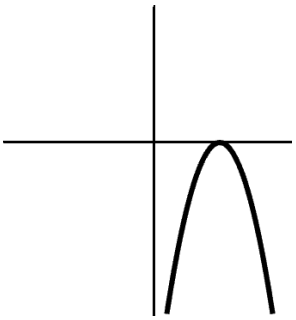
62. A quadratic function has zeros of 1 and 7. It also has an initial value of -7. This function has a maximum of 9 which occurs at $x = 4$. Sketch a graph of the function on the grid below by plotting its intercepts and vertex.



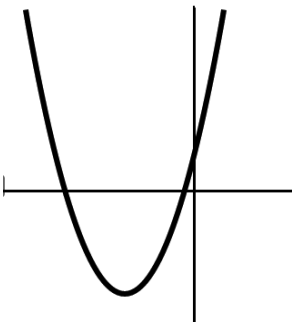
63. The following are graphs of the function $f(t) = m + n(t+q)^2$ with constants $m, n, q \in \mathbb{R}$ are given below. Determine which of the following statements below each graph are true about the values for each constants for the function graphed above.



◯ $m < 0$	◯ $n < 0$	◯ $q < 0$
◯ $m = 0$	◯ $n = 0$	◯ $q = 0$
◯ $m > 0$	◯ $n > 0$	◯ $q > 0$

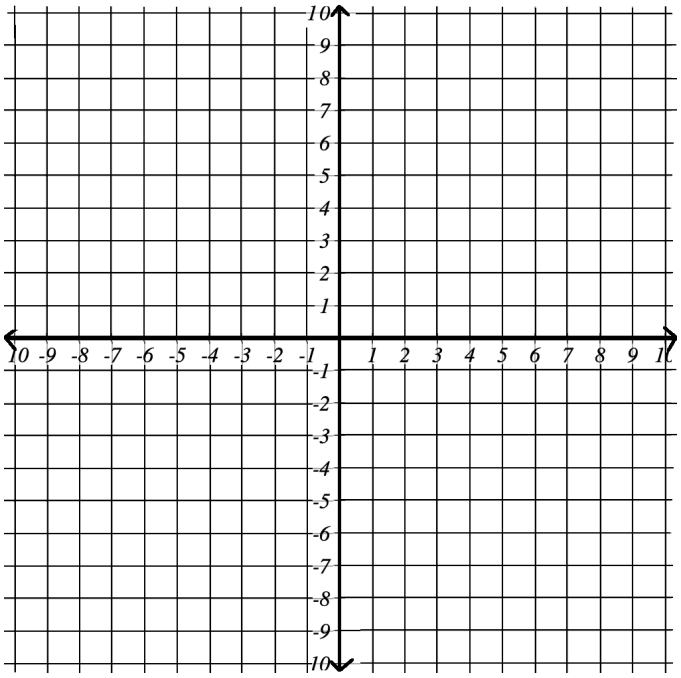


◯ $m < 0$	◯ $n < 0$	◯ $q < 0$
◯ $m = 0$	◯ $n = 0$	◯ $q = 0$
◯ $m > 0$	◯ $n > 0$	◯ $q > 0$

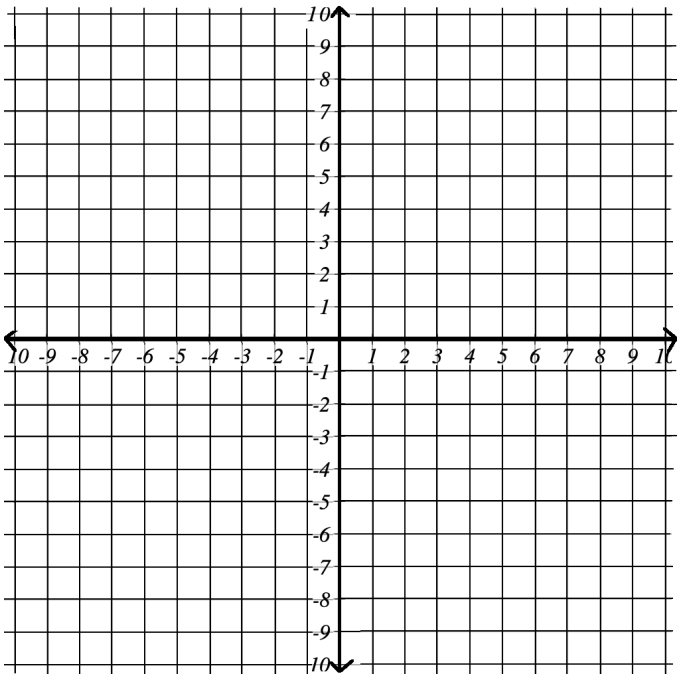


◯ $m < 0$	◯ $n < 0$	◯ $q < 0$
◯ $m = 0$	◯ $n = 0$	◯ $q = 0$
◯ $m > 0$	◯ $n > 0$	◯ $q > 0$

64. Graph the function $f(x) = \begin{cases} -5 & x < 0 \\ x - 5 & x \geq 0 \end{cases}$.

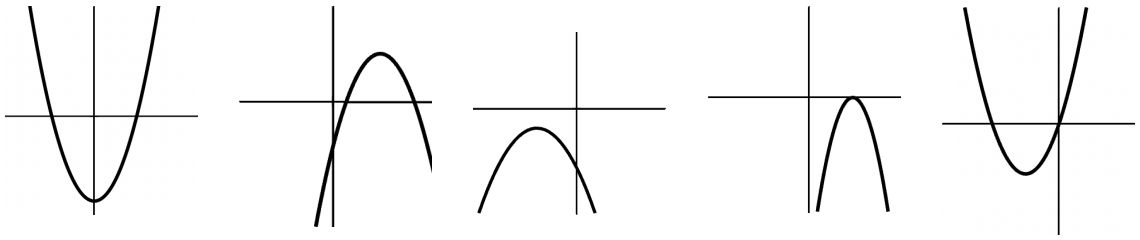


65. Graph the function $f(x) = \begin{cases} -\frac{4}{3}x - 5 & x < -6 \\ \frac{1}{3}x - 3 & x \geq -6 \end{cases}$.

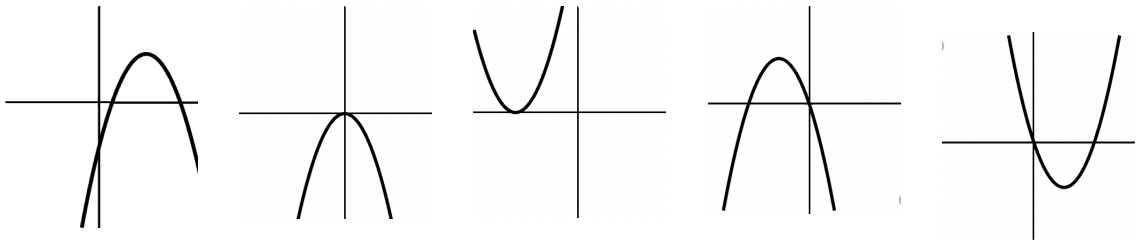


66. Carefully examine each the equation for each of the following quadratic functions. Circle all graphs that represent a graph of that function based on its equation and the constraints given for the constants in its equation.

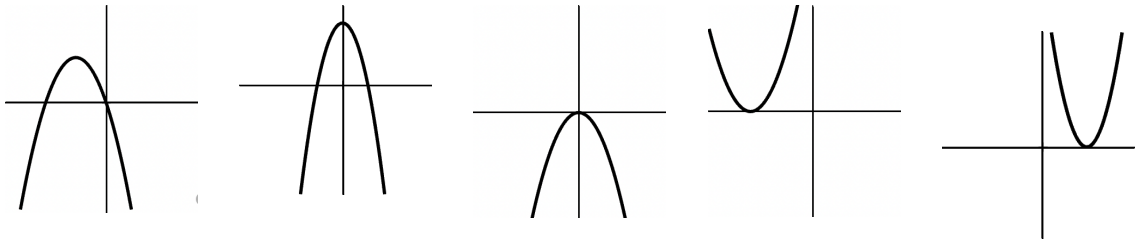
(a) $f(x) = a(x+b)(x+c)$ where $c \neq b$, and $a, b, c \neq 0$



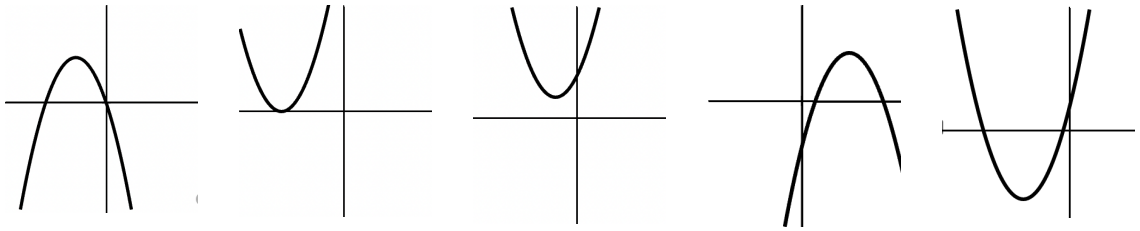
(b) $f(x) = ax - x^2, a \neq 0$



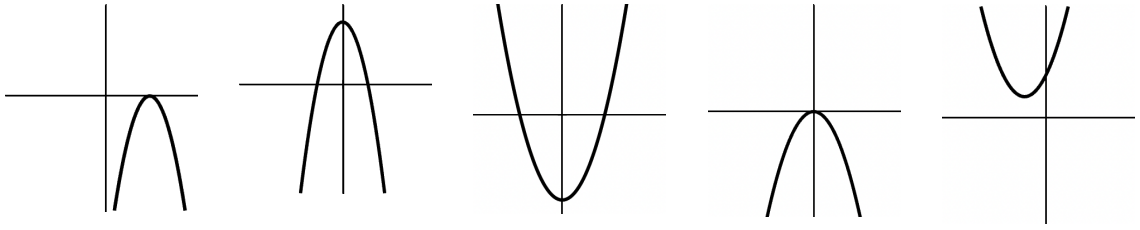
(c) $f(x) = a(x+b)^2$ where $a \neq 0$



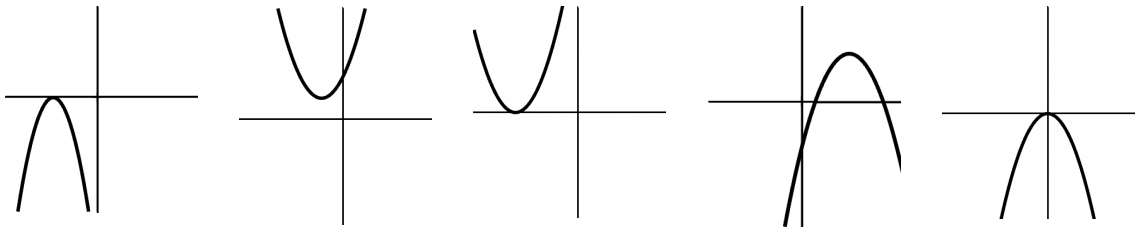
(d) $f(x) = a(x+b)^2 + c$ where $a > 0$ and $b > 0$



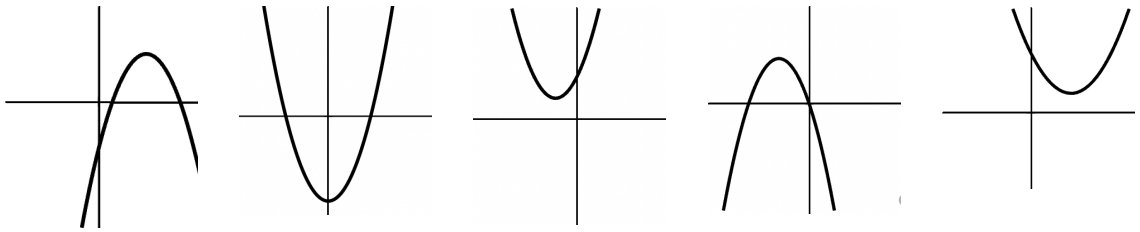
(e) $f(x) = ax^2 + b$ where $a \neq 0$



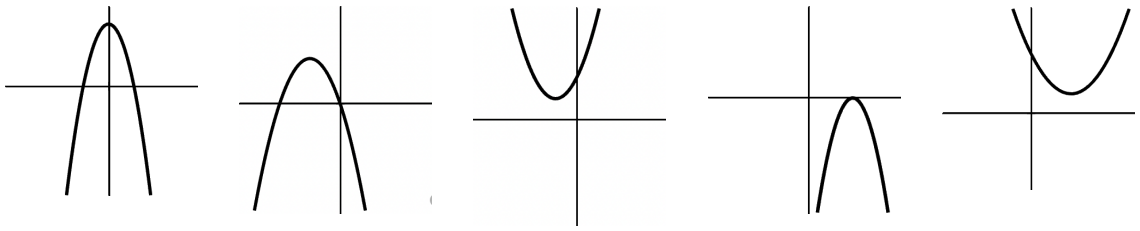
(f) $f(x) = ax^2 + bx + c$ where $ax^2 + bx + c = 0$ has one real solution.



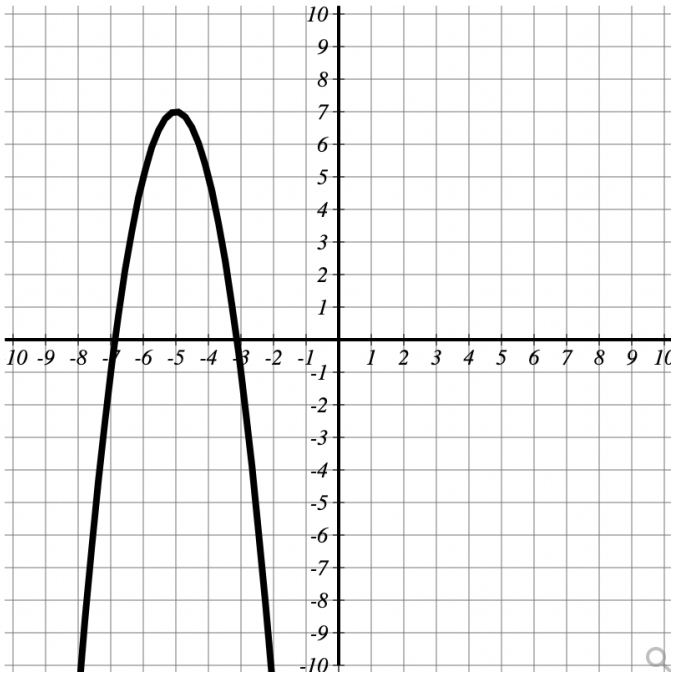
(g) $f(x) = ax^2 + bx + c$ where $c < 0$



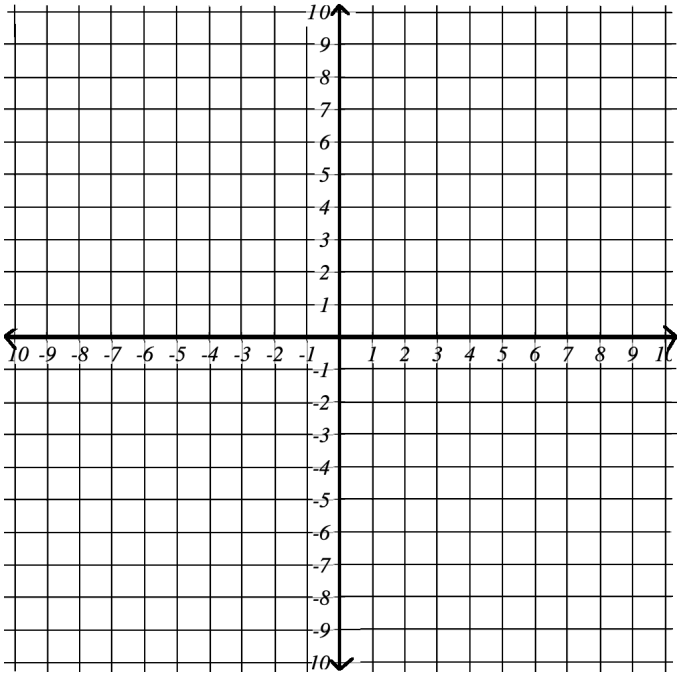
(h) $f(x) = ax^2 + bx + c$ where $-\frac{b}{2a} > 0$



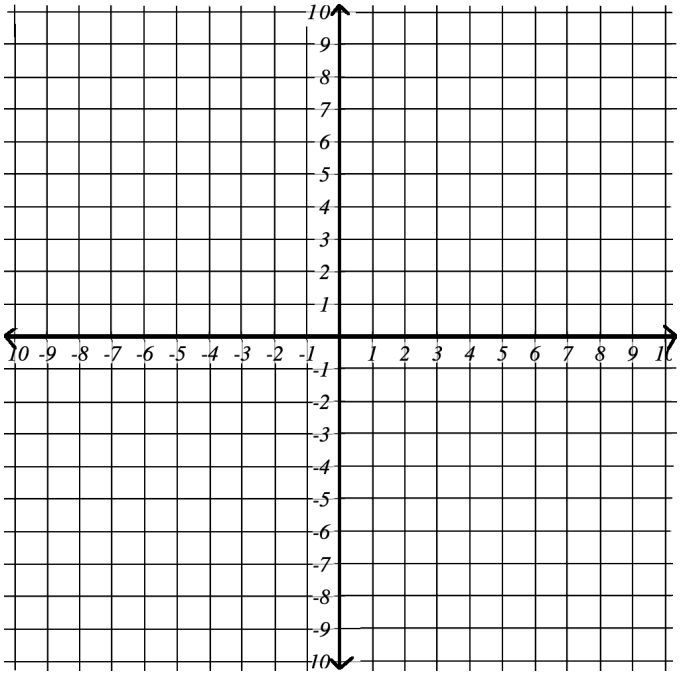
67. Write an equation for the function graphed below.



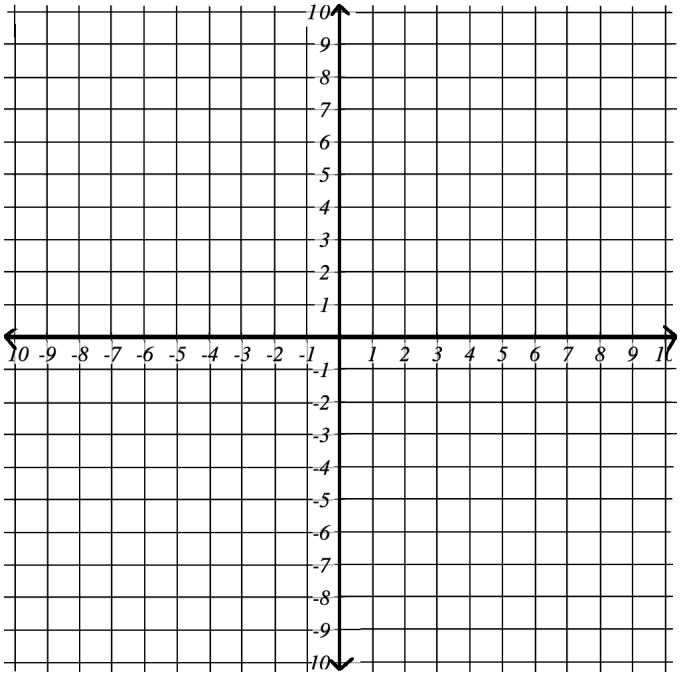
68. Graph the function $f(x) = -3 + \sqrt{x+9}$.



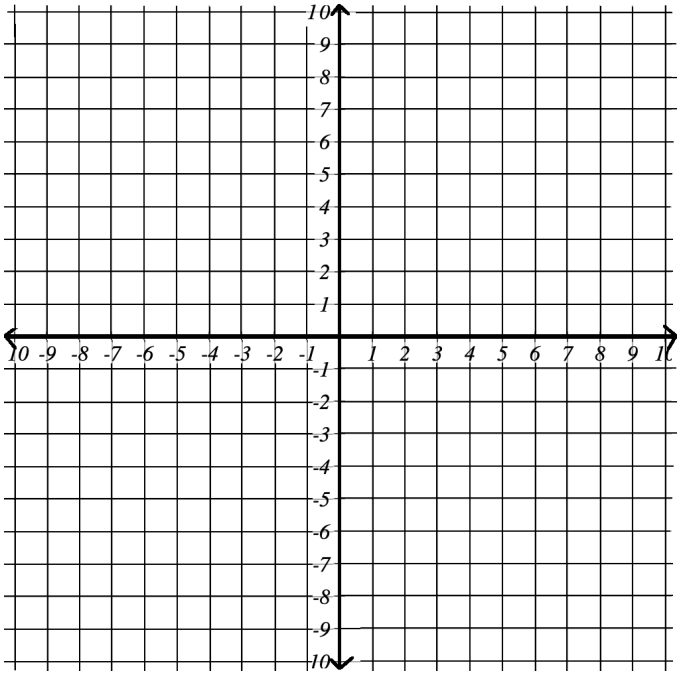
69. Graph the function $f(x) = -(x-3)^3 + 2$.



70. Graph the function $f(x) = -(x-3)^4 + 9$.



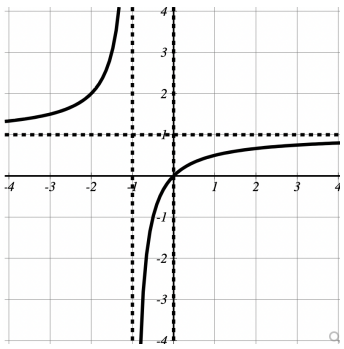
71. Graph the function $f(x) = 3\sqrt{x} - 5$.



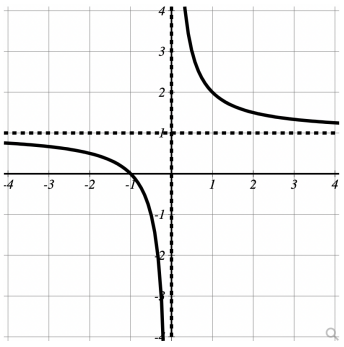
72. Match each of the following equations on the right with the graphs on the left by writing the appropriate Roman numeral in the blank.

GRAPHS:

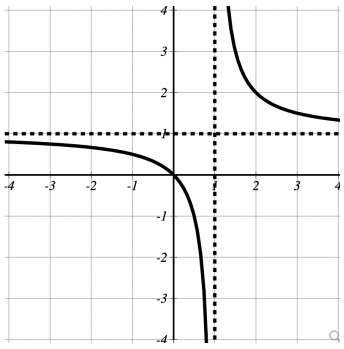
(a) _____



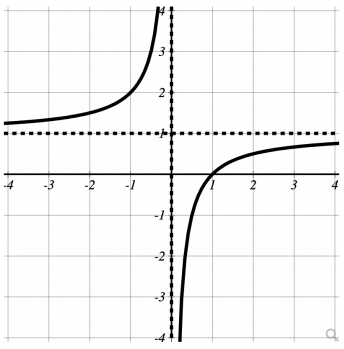
(b) _____



(c) _____



(d) _____



EQUATIONS:

(I) $y = \dfrac{x-1}{x}$

(II) $y = \dfrac{x+1}{x}$

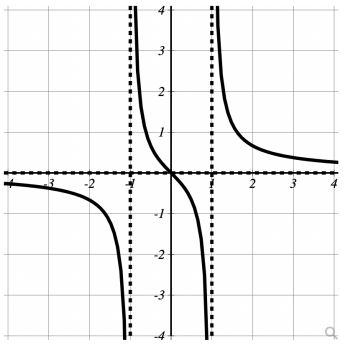
(III) $y = \dfrac{x}{x-1}$

(IV) $y = \dfrac{x}{x+1}$

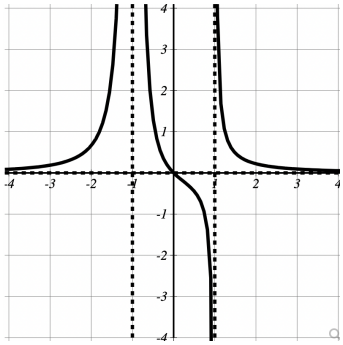
73. Match each of the following equations on the right with the graphs on the left by writing the appropriate Roman numeral in the blank. One of the equations will not be used.

GRAPHS:

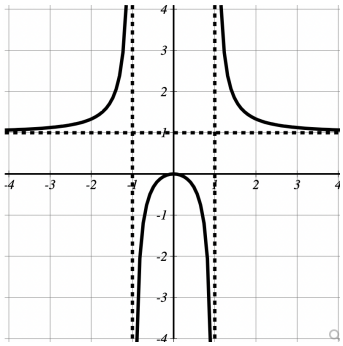
(a) _____



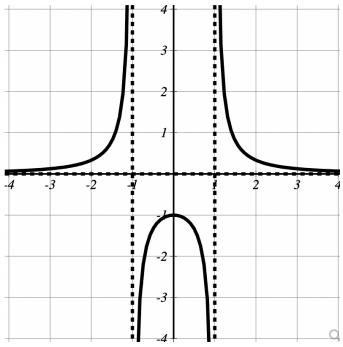
(b) _____



(c) _____



(d) _____



EQUATIONS:

(I) $y = \dfrac{1}{(x+1)(x-1)}$

(II) $y = \dfrac{x}{(x+1)(x-1)}$

(III) $y = \dfrac{x^2}{(x+1)(x-1)}$

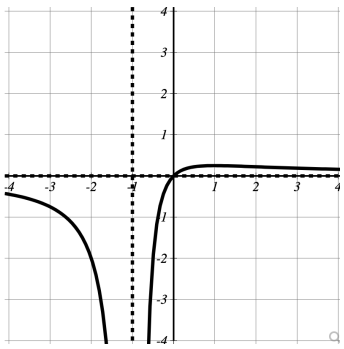
(IV) $y = \dfrac{x}{(x+1)(x-1)^2}$

(V) $y = \dfrac{x}{(x+1)^2(x-1)}$

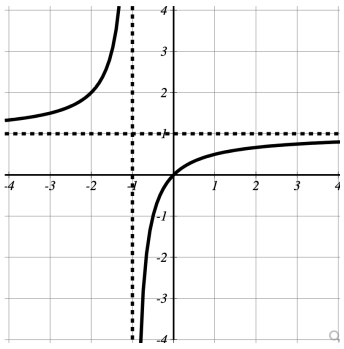
74. Match each of the following equations on the right with the graphs on the left by writing the appropriate Roman numeral in the blank.

GRAPHS:

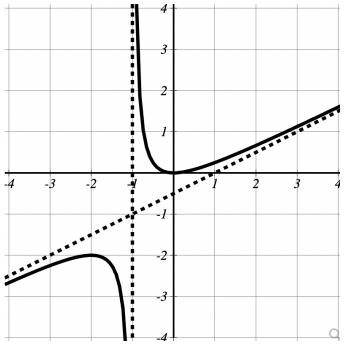
(a) _____



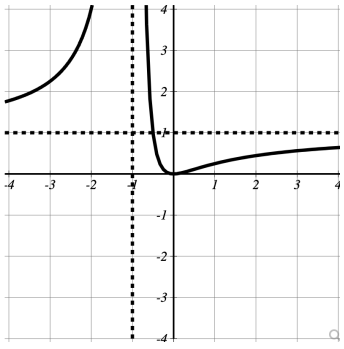
(b) _____



(c) _____



(d) _____



EQUATIONS:

(I) $y = \dfrac{x}{x+1}$

(II) $y = \dfrac{x}{(x+1)^2}$

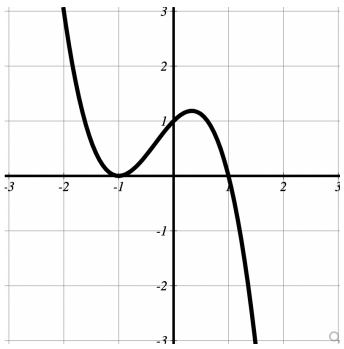
(III) $y = \dfrac{x^2}{x+1}$

(IV) $y = \dfrac{x^2}{(x+1)^2}$

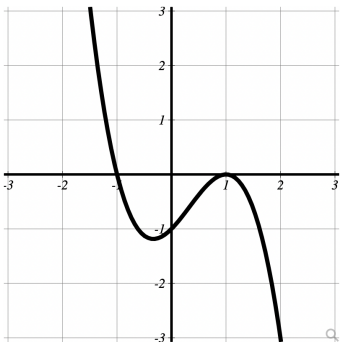
75. Match each of the following equations on the right with the graphs on the left by writing the appropriate Roman numeral in the blank. There will be four unused equations.

GRAPHS:

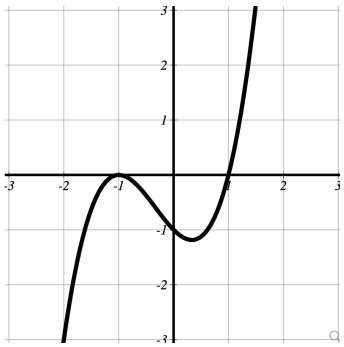
(a) _____



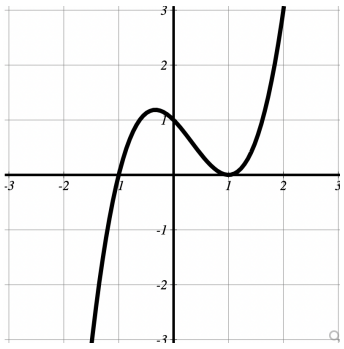
(b) _____



(c) _____



(d) _____



EQUATIONS:

(I) $y = (x+1)(x-1)$

(II) $y = (x+1)^2(x-1)$

(III) $y = (x+1)(x-1)^2$

(IV) $y = (x+1)^2(x-1)^2$

(V) $y = -(x+1)(x-1)$

(VI) $y = -(x+1)^2(x-1)$

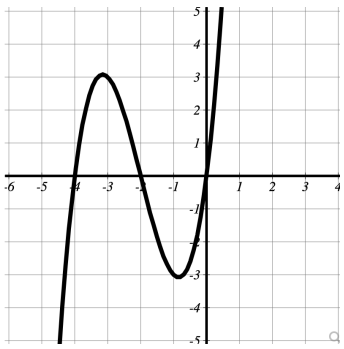
(VII) $y = -(x+1)(x-1)^2$

(VIII) $y = -(x+1)^2(x-1)^2$

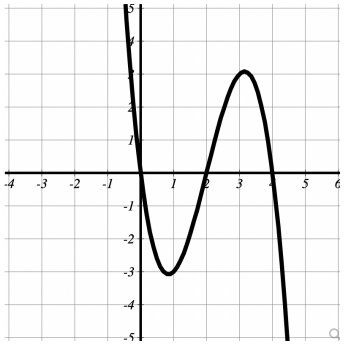
76. Match each of the following equations on the right with the graphs on the left by writing the appropriate Roman numeral in the blank.

GRAPHS:

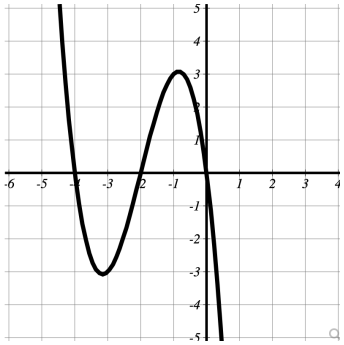
(a) _____



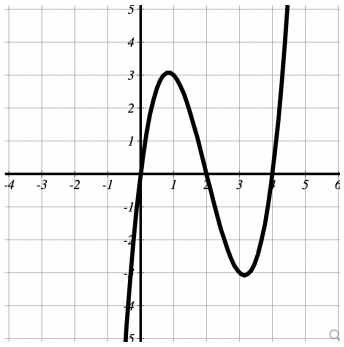
(b) _____



(c) _____



(d) _____



EQUATIONS:

(I) $y = x(x+2)(x+4)$

(II) $y = -x(x+2)(x+4)$

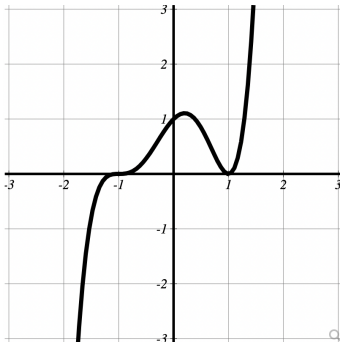
(III) $y = x(x-2)(x-4)$

(IV) $y = -x(x-2)(x-4)$

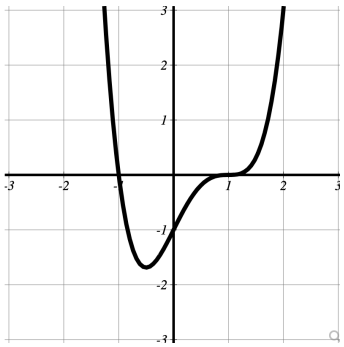
77. Match each of the following equations on the right with the graphs on the left by writing the appropriate Roman numeral in the blank. Two equations will not be used

GRAPHS:

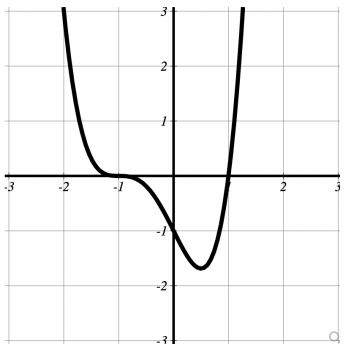
(a) _____



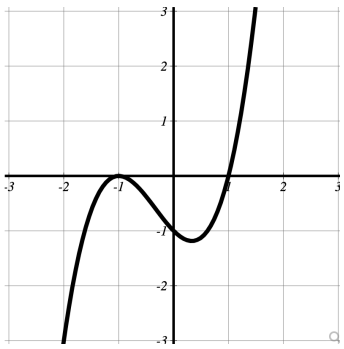
(b) _____



(c) _____



(d) _____



EQUATIONS:

(I) $y = (x+1)(x-1)^3$

(II) $y = (x+1)^3(x-1)$

(III) $y = (x+1)(x-1)^2$

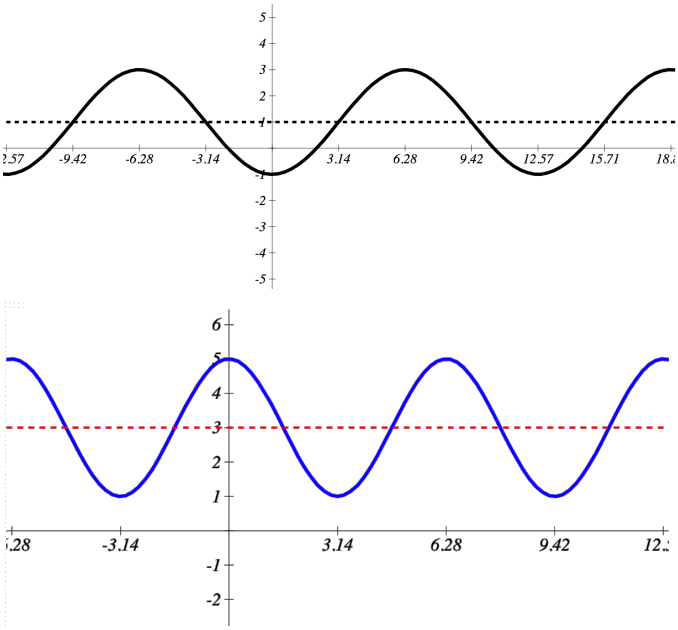
(IV) $y = (x+1)^2(x-1)$

(V) $y = (x+1)^2(x-1)^3$

(VI) $y = (x+1)^3(x-1)^2$

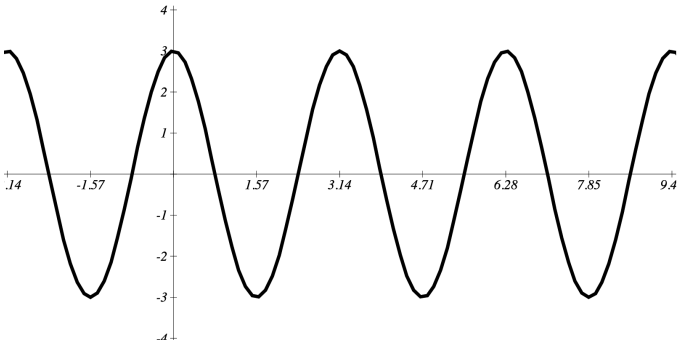
78. Let $h(x) = 3 - \frac{1}{2}\sin(10x - \pi)$. Describe each of the following.
    (a) What is the amplitude of h?
    (b) What is the period of h?
    (c) How much is h shifted up or down?
    h is shifted (circle one) up/down _____ units.
    (d) How much is h shifted left or right?
    h is shifted (circle one) left/right _____ units.

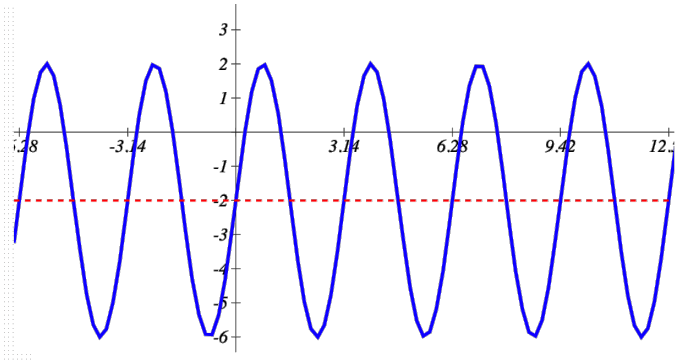
79. Write an equation for each of the following functions. Hint: you can write an equation for each of these without a horizontal shift.



80. Let $h(x) = 3 - 5\sin(\frac{x}{4} + \pi)$. Describe each of the following.
    (a) What is the amplitude of h?
    (b) What is the period of h?
    (c) How much is h shifted up or down?
    h is shifted (circle one) up/down _____ units.
    (d) How much is h shifted left or right?
    h is shifted (circle one) left/right _____ units.

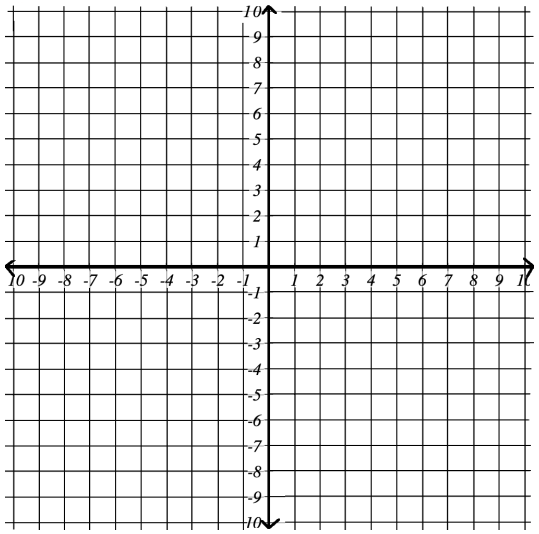
81. Write an equation for each of the following functions. Hint: you can write an equation for each of these without a horizontal shift.



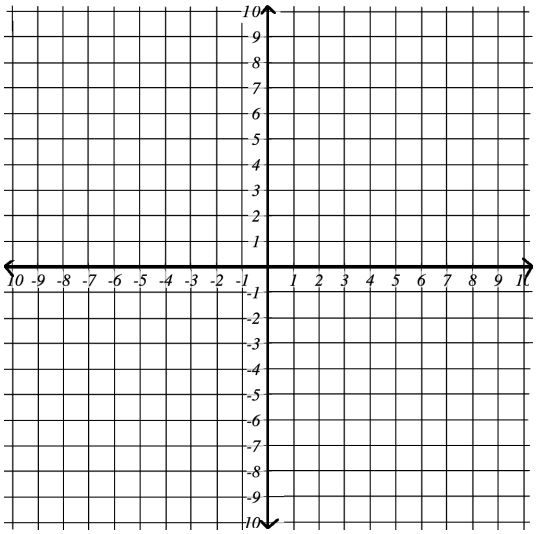


82. Graph the following functions.

(a) $y = 5 - 2^{x+4}$



(b) $y = \log_3(-x)$

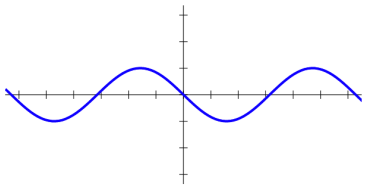


83. Match each equation with its graph. Write the letter for the graph in the blank next to the appropriate equation.

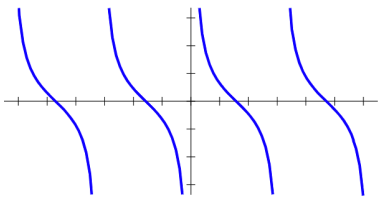
$f(x) = \cos(x)$ ____ $f(x) = \csc(x)$ ____ $f(x) = \sec(x)$ ____

$f(x) = \cot(x)$ ____ $f(x) = \sin(x)$ ____ $f(x) = \tan(x)$ ____

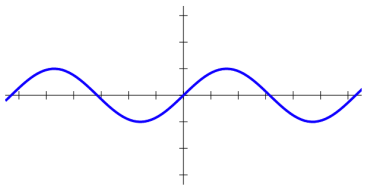
A.



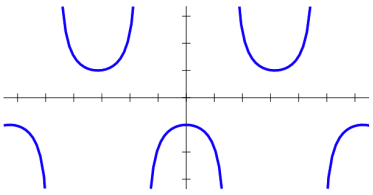
B.



C.



D.



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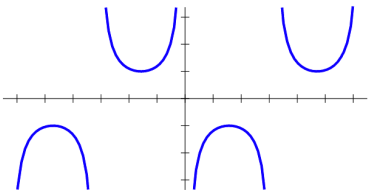
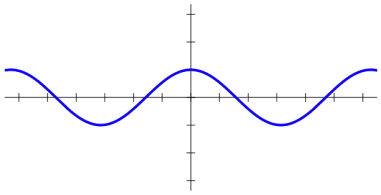
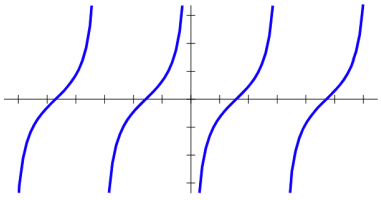


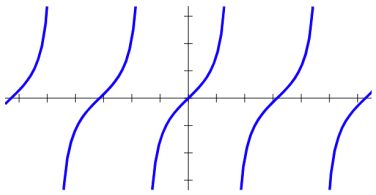
F.



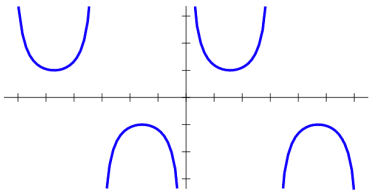
G.



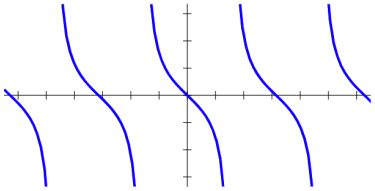
H.



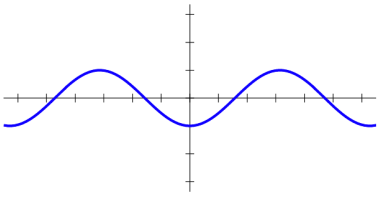
I.



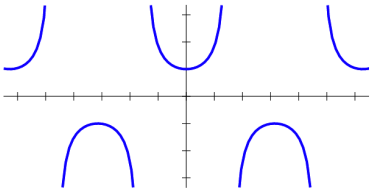
J.



K.



L.



Rewriting Equations of Functions

84. Rewrite each of the following function's equations in factored form.
    (a) $g(t) = 75t^2 - 3$
    (b) $f(x) = x^2 + 4x - 12$
    (c) $g(n) = 5n^2 + 11n + 2$

85. Complete the square to rewrite $h(x) = x^2 - 6x + 5$ in the form $y = a(x + h)^2 + k$.

86. Complete the square to rewrite $f(x) = 2x^2 + 8x + 1$ in the form $y = a(x + h)^2 + k$.

87. Expand and simplify $g(x) = 3(2x - 1)^2$ to rewrite in the form $y = ax^2 + bx + c$.

88. Rewrite each of the following in factored form.
    (a) $f(x) = 4x^2 - 9$
    (b) $g(t) = 6t^5 - 14t^4$
    (c) $k(a) = a^2 + 2a - 24$
    (d) $g(x) = 5x^2 + 13x - 6$

89. Fully simplify $k(m) = \dfrac{2m^2 - 8}{m^2 + m - 2}$. Write any needed restriction in the domain.

90. Rewrite each of the following in the form $y = ax^n$.
    (a) $f(x) = 5\sqrt{x}$
    (b) $h(t) = \dfrac{1}{9t^3}$
    (c) $r(a) = \dfrac{7}{8\sqrt[5]{a^3}}$

91. Fully simplify $g(a) = \dfrac{a^2 - 5a - 36}{2a^2 + 18a + 40}$. Include any needed restriction in the domain.

92. Rewrite each of the following in the form $y = ax^n$.
    (a) $r(z) = \dfrac{7}{\sqrt[3]{z^5}}$
    (b) $f(x) = \dfrac{\sqrt{x}}{8}$
    (c) $h(t) = \dfrac{5}{9t^3}$

93. Simplify the equation of each of the following functions fully.
    (a) $f(x) = \tan x \csc x$
    (b) $g(x) = \cot x \cos x \sin x + \sin^2 x$
    (c) $k(x) = \tan^2 x \cos^2 x - \tan x \cot x$

94. Simplify the equation of each of the following functions fully.
    (a) $f(x) = \sin^2 x \csc x \sec x$
    (b) $g(x) = \tan x \cot x - \dfrac{1}{\csc^2 x}$
    (c) $k(x) = \tan^2 x \csc^2 x - 1$

95. Rewrite the equation of g by combining the logarithms
    $k(t) = \log_7(t + 5) - 5\log_7(t)$

96. Rewrite the equation of g by expanding the logarithms and simplifying fully
    $g(n) = \log_2\left(\dfrac{16n^7}{n + 1}\right)$

Function Algebra

97. Let $h(x) = 4x - 3$ and $j(x) = 10x^2$. Find each of the following and write the equation in simplified and in descending order.
    (a) $(j \circ h)(x)$
    (b) $(h \circ j)(x)$

98. Let $f(x) = 5x^2 - 2x + 3$ and $g(x) = 2x - 7$. Find a simplified equation, written in descending order, for the function $f \circ g$.

99. Let $m(x) = x^2 + x - 1$ and $n(x) = 2x^2 - x + 3$. Find a simplified equation, written in descending order, for each of the following functions.
    (a) $(m \cdot n)(x)$
    (b) $(m - n)(x)$
    (c) $(5m)(x)$
    (d) $(m + n)(-1)$

100. Let $f(x) = 4 + \sqrt[3]{x + 5}$. Find the equation for f^{-1}.

101. Assume $f(x) = 6x^2$, $g(x) = 8x^4$, and $k(x) = 2x^{-5}$. Find each of the following. Write your final answers simplified with no negative exponents. Make sure that your final answer has the correct domain.
    (a) $(f \cdot g)(x)$
    (b) $(\frac{f}{g})(x)$
    (c) $(\frac{g}{k})(x)$
    (d) $(f \circ k)(x)$

102. Let $f(x) = 5 - \dfrac{\sqrt[3]{2x + 1}}{4}$. Find the equation for f^{-1}.

103. Let $f(x) = 5x^4$ and $g(x) = 10x^2$. Find the equation of $g \circ f$ and simplify it fully.

104. Let $k(x) = (x + 3)(x - 4)$ and $h(x) = x(x - 4)$. Find the equation of $\dfrac{h}{k}$ and simplify it fully. Include any needed restriction in the domain.

105. Let $p(x) = 2ax^5$ and $q(x) = a^3x^4$ where $a \in \mathbb{R}$ is a constant. Find the equation of $p \cdot q$ and simplify it fully.

106. Let $y(w) = w^{1/2}$ and $r(w) = \dfrac{a^4w^{-8}}{9b^2}$ for $a, b \in \mathbb{R}$. Find the equation of $y \circ r$. Write the equation fully simplified without any rational or negative exponents.

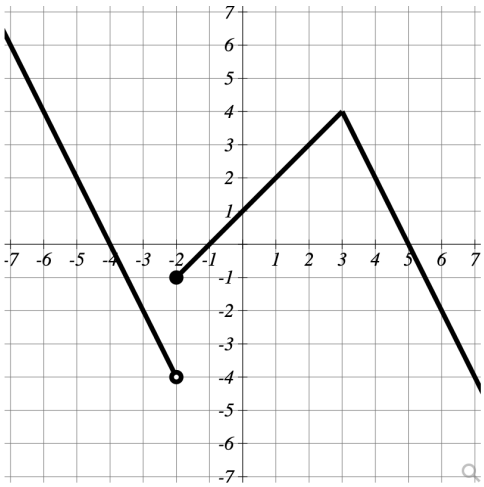
107. Find the inverse of $f(x) = -4 + \frac{2}{3}(10^{x+5})$.

108. Find the inverse of $f(x) = 7 + 2\ln(x + 8)$.

109. Find the inverse of $k(t) = \dfrac{1 - 3^x}{9}$.

110. The functions f and g are defined below.

x	-1	0	1	2	3
$f(x)$	9	6	3	0	-3



Graph of $g(x)$.

Find each of the following. Make sure you pay careful attention to what is being asked and which function each question is referring to.

(a) Find $(f \cdot g)(1)$

(b) Find $(g \circ f)(0)$

111. The function $g(x)$ and $h(x)$ are defined by the tables below:

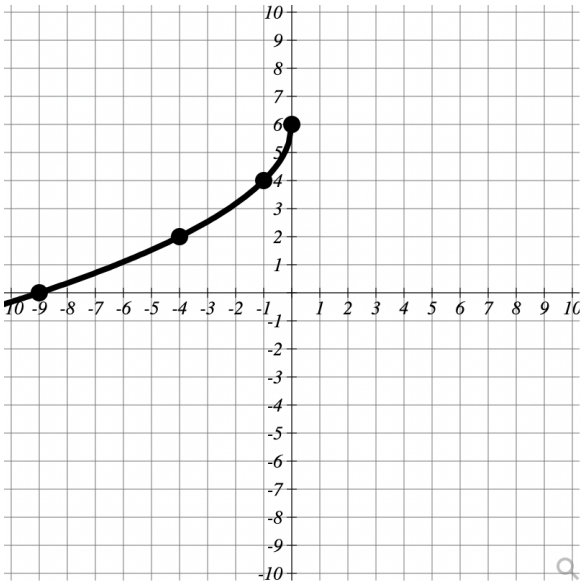
x	1	2	3	4	5	6	7	8
$g(x)$	11	9	7	5	3	1	-1	-3

x	-1	0	1	2	3	4	5	6
$h(x)$	-3	4	10	15	19	22	24	25

(a) Find the value of $(g \circ h)(0)$

(b) Find $g^{-1}(3)$.

112. The function $f(x)$ is graphed on the grid below. Draw the graph of $f^{-1}(x)$ on the same grid.



113. A table is given below for the function $h(x)$. Also use the graph in the previous problem as a definition for the function $f(x)$.

x	-1	0	1	2	3	4	5	6
$h(x)$	-3	3	5	4	2	0	1	-1

Find each of the following.

(a) $h^{-1}(2)$

(b) The zeros of h^{-1}.

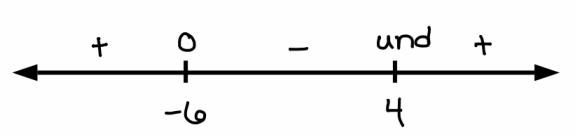
(c) $(f+h)(-1)$

(d) $(h \circ f)(-4)$

Inequalities and Functions

114. Create a sign diagram for the function $f(x) = (x+3)(x-4)^2$.

115. The sign diagram for $f(x) = \dfrac{x+6}{x-4}$ is given below:



Find each of the following using the sign diagram:

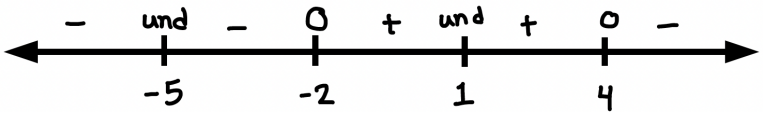
(a) What is the domain of $k(x) = \sqrt{\dfrac{x+6}{x-4}}$?

(b) What is the domain of $k(x) = \ln\left(\dfrac{x+6}{x-4}\right)$?

(c) Where is the function $f(x) = \dfrac{x+6}{x-4}$ below the x-axis?

116. Create a sign diagram for the function $f(x) = \dfrac{2-x}{x+5}$.

117. The sign diagram for $f(x) = \dfrac{x^2-2x-8}{x^4+8x^3+6x^2-40x+25}$ is given below:



Find each of the following using the sign diagram:

(a) What is the domain of $k(x) = \sqrt{\dfrac{x^2-2x-8}{x^4+8x^3+6x^2-40x+25}}$?

(b) What is the domain of $m(x) = \log_2\left(\dfrac{x^2-2x-8}{x^4+8x^3+6x^2-40x+25}\right)$?

(c) Solve $\dfrac{x^2-2x-8}{x^4+8x^3+6x^2-40x+25} \leq 0$.

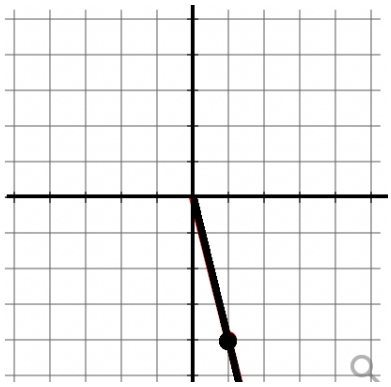
(d) Where is the function $f(x) = \dfrac{x^2-2x-8}{x^4+8x^3+6x^2-40x+25}$ below the x-axis?

(e) Solve $\dfrac{x^2-2x-8}{x^4+8x^3+6x^2-40x+25} > 0$.

Transcendental Operations

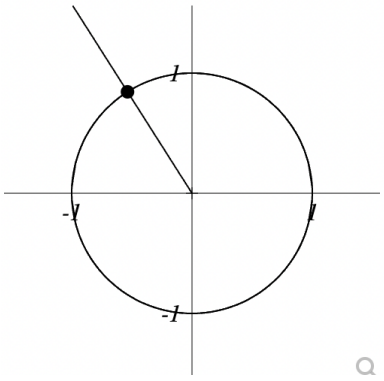
118. Let $\phi = \dfrac{16\pi}{11}$. Give the following answers in radians.
   (a) Draw ϕ in standard position.
   (b) State all angles that are coterminal with ϕ.
   (c) State one angle that is coterminal with ϕ.
   (d) State the reference angle of ϕ.

119. The angle α is graphed in standard position on the grid below. The point $(1, -4)$ lays on the terminal side of α.



   Find the value of each of the following trig functions:
   (a) $\sin\alpha =$
   (b) $\cos\alpha =$
   (c) $\tan\alpha =$

120. The angle θ intersects the edge of the unit circle at the point $(-0.5361, 0.8442)$.



   Find the value of each of the following trig functions:
   (a) $\sin\theta =$
   (b) $\cos\theta =$

121. Assume $\sin\beta = -\dfrac{2}{5}$ and $\cos\beta = -\dfrac{\sqrt{21}}{5}$. Find the values of each of the following trig functions:
   (a) $\sec\beta =$

(b) $\csc\beta =$

(c) $\cot\beta =$

122. Evaluate each of the following:

(a) $\sin\frac{2\pi}{3} =$

(b) $\cos\pi =$

(c) $\sin\frac{7\pi}{6} =$

(d) $\cos\frac{7\pi}{4} =$

(e) $\tan\frac{\pi}{4} =$

(f) $\cot\frac{\pi}{2} =$

(g) $\sec\frac{4\pi}{3} =$

(h) $\arcsin(\sin\frac{5\pi}{6}) =$

(i) $\arctan(\sin\frac{3\pi}{2}) =$

(j) $\arccos(\cos\frac{5\pi}{4}) =$

123. Rewrite $6 = 7^{x+5}$ in logarithmic form.

124. Rewrite $8 = \log(2x+3)$ in exponential form.

125. Let $f(x) = \log_2 x$. Find the value of each of the following.

(a) $f(\frac{1}{4})$

(b) $f(32)$

(c) $f(1)$

(d) $f(2)$

(e) $f(\frac{1}{2})$

(f) $f(-2)$

(g) $f(0)$

(h) $f(2)$

126. Let $\theta = \dfrac{7\pi}{10}$.

(a) Draw θ in standard position.

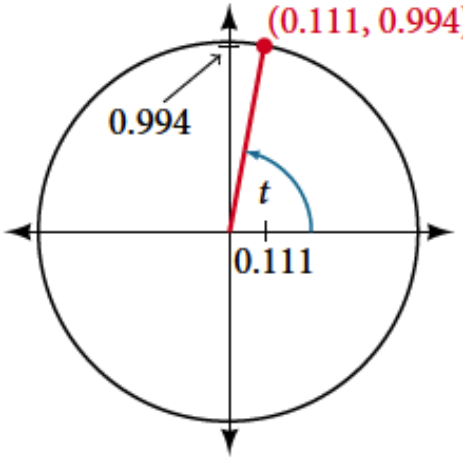
(b) Give one angle that is coterminal with θ.

(c) State all angles that are coterminal with θ.

(d) State the reference angle of θ.

127. ABC is a right triangle with $C = 90°$. If $A = 17°$ and $a = 19.6$, find the exact value of b.

128. The angle t is graphed in standard position below. The point where t intersects the edge of the unit circle is marked with its coordinates below.

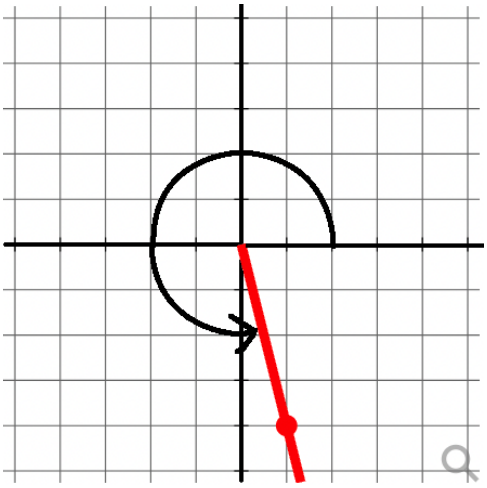


State the value of the following:

(a) $\sin(t) =$

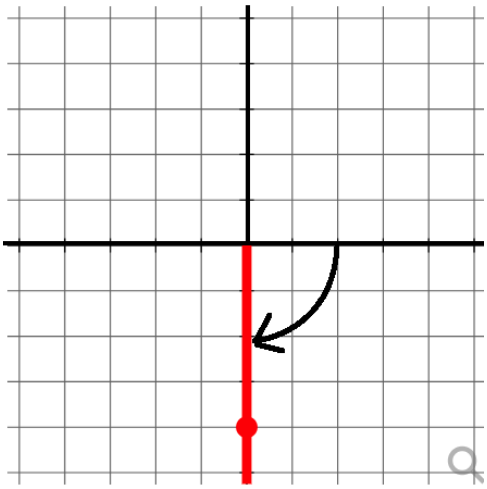
(b) $\cos(t) =$

129. The angle α is graphed below. Give the value of all six trig functions.



(a) $\sin\alpha$
(b) $\cos\alpha$
(c) $\tan\alpha$
(d) $\cot\alpha$
(e) $\sec\alpha$
(f) $\csc\alpha$

130. The angle β is graphed below. Give the value of all six trig functions.



(a) $\sin\beta$
(b) $\cos\beta$
(c) $\tan\beta$
(d) $\cot\beta$
(e) $\sec\beta$
(f) $\csc\beta$

131. Find the exact value of each of the following. Rationalize the denominator and simplify completely if needed.

(a) $\sin(\frac{\pi}{3})$
(b) $\cos(\frac{5\pi}{4})$
(c) $\sin(\frac{11\pi}{6})$
(d) $\csc(2\pi)$
(e) $\tan(\frac{2\pi}{3})$
(f) $\cos(\frac{\pi}{2})$
(g) $\tan(\frac{7\pi}{4})$
(h) $\sec(\pi)$
(i) $\cot(\frac{4\pi}{3})$
(j) $\csc(\frac{7\pi}{4})$