Unit 3 Assessment C

Name: Answer Key

Learning Objective	Grade
Rates of Change	
Pointwise Behavior	
Global Behavior	
Graphing	
Rewriting Equations	
Transcendental Operations	

Functions as Quantities Changing with Each Other

40pts

1. Find the average rate of change of $g(\theta) = \cos\theta$ between $\theta = \pi$ and $\theta = 2\pi$.

$$ARC = \frac{g(2\pi) - g(\pi)}{2\pi - \pi} = \frac{\cos(2\pi) - \cos(\pi)}{\pi}$$

$$= \frac{1 - (-1)}{\pi}$$

$$= \boxed{\frac{2}{\pi}}$$

60pts

2. The angle θ is sweeping through Quadrant III in the **positive** direction. Describe each of the following:

(a) $\sin\theta$ is...
- ○ Positive
- ● Negative
- ○ Zero

(b) $\sin\theta$ is...
- ○ Increasing
- ● Decreasing
- ○ Constant

(c) The rate at which $\sin\theta$ is changing is...
- ○ Positive
- ● Negative
- ○ Zero

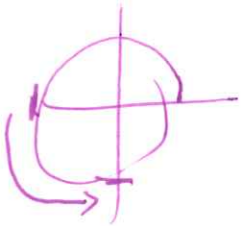
decreasing = neg. r.o.c.

(d) $\cos\theta$ is...
- ○ Positive
- ● Negative
- ○ Zero

(e) $\cos\theta$ is...
- ● Increasing
- ○ Decreasing
- ○ Constant

(f) The rate at which $\cos\theta$ is changing is...
- ● Positive
- ○ Negative
- ○ Zero

increasing = pos. r.o.c.



Behavior of Function at a Point

3. Let $f(x) = \sqrt{2} + 2\cos x$

10 (a) Find the y-intercept of f.

$f(0) = \sqrt{2} + 2\cos 0$

$= \sqrt{2} + 2 \cdot 1$

$= \sqrt{2} + 2$

$(0, \sqrt{2}+2)$

30 (b) Find the all zeros of f in $(-\infty, \infty)$.

$\sqrt{2} + 2\cos x = 0$

$\cos x = -\frac{\sqrt{2}}{2}$

$x_t = \arccos\left(-\frac{\sqrt{2}}{2}\right)$

$= \frac{3\pi}{4}$

($\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, now do horiz. flip to get $-\frac{\sqrt{2}}{2}$)

$x_b = -x_t = -\frac{3\pi}{4}$

$x = \pm\frac{3\pi}{4} + 2\pi K$, K is an integer

4. Find the value of θ in each of the following situations.

20 (a) $\theta = \arcsin(-\frac{\sqrt{3}}{2})$

$\theta = -\frac{\pi}{3}$

$\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

↓ vert. flip to get $-\frac{\sqrt{3}}{2}$

$\sin(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

20 (b) $\sin\theta = -\frac{\sqrt{3}}{2}$ where θ is in $[0, 2\pi]$

$\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

$\theta = -\frac{\pi}{3}$ is one solution, but not on $[0, 2\pi]$

$\theta = \pi - (-\frac{\pi}{3}) = \frac{4\pi}{3}$ is another solution

$-\frac{\pi}{3} + 2\pi = \frac{5\pi}{3}$ is a solution

20 (c) $\sin\theta = -\frac{\sqrt{3}}{2}$ where θ is in $(-\infty, \infty)$

Just need general solutions:

$\theta = \frac{4\pi}{3} + 2\pi K$, K is an integer

or $\frac{5\pi}{3} + 2\pi K$

Behavior of Function Over an Interval

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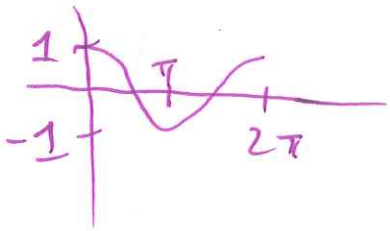
5. Let $g(x) = \cos x$, $(0 \leq x \leq 2\pi)$. Answer the following questions about the function g. Drawing a quick sketch of g may help you answer these questions.

   (a) What is the range of g?

   $[-1, 1]$



   (b) Over what interval is g increasing?

   $(\pi, 2\pi)$

   (c) Over what interval is g decreasing?

   $(0, \pi)$

   (d) Over what interval is g concave up?

   $(\frac{\pi}{2}, \frac{3\pi}{2})$

   (e) Over what interval is g concave down?

   $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$

   (f) Describe the extrema of g

   The function g has an absolute maximum of 1 *at* $x =$ 0 and 2π.

   The function g has an absolute minimum of -1 *at* $x =$ π.

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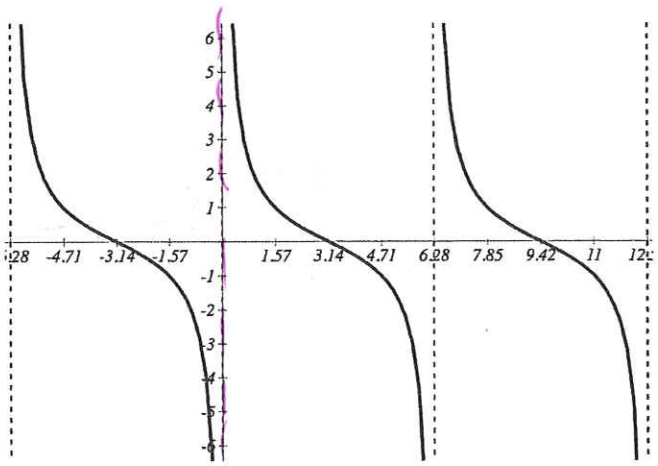
6. The graph $m(t)$ is graphed below:



Find each of the following.

(a) $\lim_{t\to 0^-} m(t) =$ $-\infty$

(b) $\lim_{t\to 0^+} m(t) =$ ∞

(c) $\lim_{t\to 0} m(t) =$ DNE

(d) What is the range of m?

$(-\infty, \infty)$

Graphs of Functions

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7. Let $h(x) = 2 - 4\sin(\frac{x}{5} + \pi)$. Describe each of the following. $h(x) = 2 - 4\sin(\frac{1}{5}(x + 5\pi))$

(a) What is the amplitude of h?

4

(b) What is the period of h?

$\frac{2\pi}{1/5} = 5 \cdot 2\pi = 10\pi$

(c) How much is h shifted up or down?

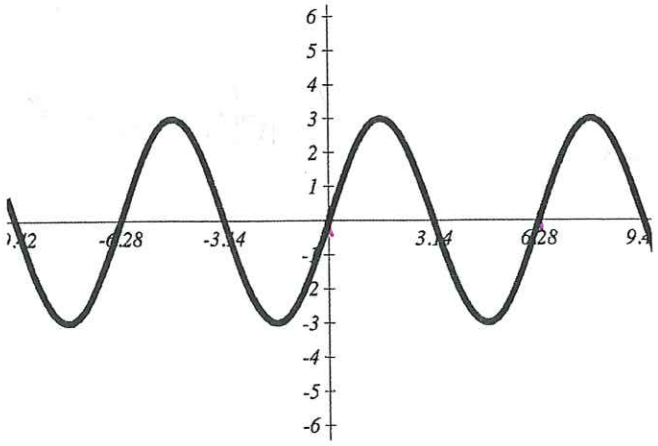
h is shifted (circle one) up/down 2 *units.*

(d) How much is h shifted left or right?

h is shifted (circle one) left/right 5π *units.*

8. Write an equation for each of the following functions. Hint: you can write an equation for each of these without a horizontal shift.

30 each



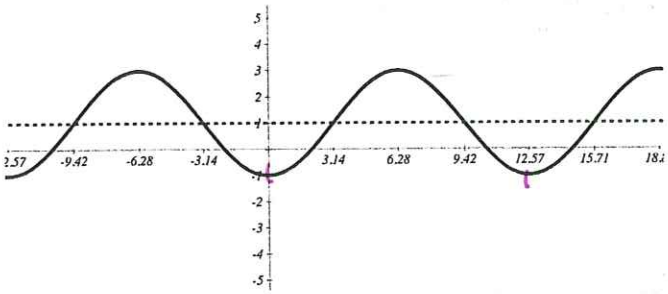
start at middle, increasing $\Rightarrow +\sin$

period $= 2\pi \Rightarrow B = 1$

Ampl $= 3$

V.S $= 0$

$f(x) = 3\sin(x)$



start at bottom $\Rightarrow -\cos$

period $= 4\pi \Rightarrow B = \frac{2\pi}{4\pi} = \frac{1}{2}$

Amp $= 2$

V.S $= 1$.

$g(x) = -2\cos\left(\frac{x}{2}\right) + 1$

Rewriting Equations of Functions

9. Simplify the equation of each of the following functions fully. Show all steps

(a) $f(x) = \cot x \sin x$

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$= \frac{\cos x}{\sin x} \cdot \sin x = \cos x \qquad (x \neq \pi K,\ K \in \mathbb{Z})$

(b) $g(x) = \tan x \cos x \sin x + \cos^2 x$

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$= \frac{\sin x}{\cos x} \cos x \sin x + \cos^2 x$

$= \sin^2 x + \cos^2 x$

$= 1 \qquad (x \neq \frac{\pi}{2} + \pi K,\ K \in \mathbb{Z})$

(c) $k(x) = \cot^2 x \sin^2 x - \tan x \cot x$

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$= \frac{\cos^2 x}{\sin^2 x} \cdot \sin^2 x - \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x}$

$= \cos^2 x - 1$

$= -\sin^2 x \qquad (x \neq \pi K,\ K \in \mathbb{Z})$

$x \neq \frac{\pi}{2} + \pi K$

Transcendental Operations

10. Let $\phi = \frac{10\pi}{7}$. Give the following answers in radians.

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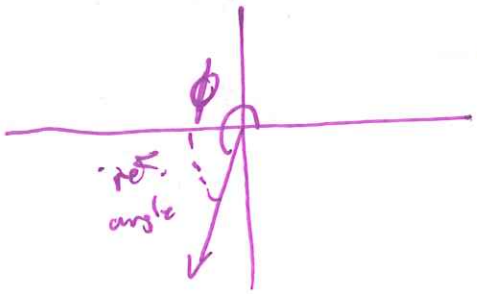
(a) Draw ϕ in standard poisition.



(b) State all angles that are coterminal with ϕ.

$\frac{10\pi}{7} + 2\pi k, \quad k \in \mathbb{Z}$

(c) State one angle that is coterminal with ϕ.

$\frac{10\pi}{7}$; P also $\frac{10\pi}{7} + 2\pi = \frac{24\pi}{7}$

(coterminal with ϕ!) and many more.

(d) State the reference angle of ϕ.

Ref. angle $= \phi - \pi = \frac{3\pi}{7}$

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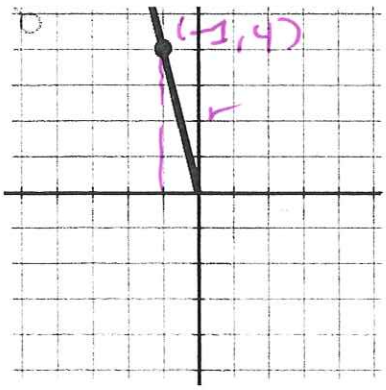
11. The angle α is graphed in standard position on the grid below. The point $(-1, 4)$ lays on the terminal side of α.



$(-1)^2+4^2=r^2$

$1+16=r^2$

$r=\sqrt{17}$

Find the value of each of the following trig functions:

(a) $\sin\alpha =$ $4/\sqrt{17}$

(b) $\cos\alpha =$ $-1/\sqrt{17}$

(c) $\tan\alpha =$ $-\frac{4}{1}=-4$

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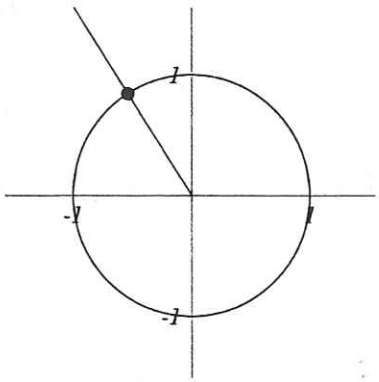
12. The angle θ intersects the edge of the unit circle at the point $(-0.5361, 0.8442)$.



Find the value of each of the following trig functions:

(a) $\sin\theta =$ 0.8442

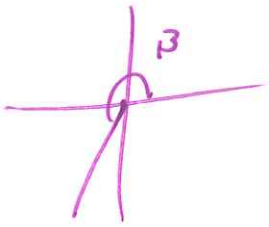
(b) $\cos\theta =$ -0.5361

should be $-\frac{\sqrt{3}}{4}$, but no penalty from don't notice & fix

15 13. Assume $\sin\beta = -\frac{\sqrt{13}}{4}$ and $\cos\beta = -\frac{3}{4}$. Find the values of each of the following trig functions:

(a) $\sec\beta =$ $\frac{1}{\cos\beta} = -\frac{4}{3}$

$\frac{\sin<0}{\cos<0} \Rightarrow$ Q3



(b) $\csc\beta =$ $\frac{1}{\sin\beta} = -\frac{4}{\sqrt{13}}$

(c) $\tan\beta =$ $\frac{\sin\beta}{\cos\beta} = \frac{-\sqrt{13}/4}{-3/4} = \frac{\sqrt{13}}{3}$

40 14. Evaluate each of the following:

(a) $\cos\pi =$ -1

(b) $\sin\frac{2\pi}{3} =$ Q2 $+\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(c) $\sin\frac{7\pi}{6} =$ Q3 $-\sin\frac{\pi}{6} = -\frac{1}{2}$

(d) $\cos\frac{7\pi}{4} =$ Q4 $+\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

(e) $\cot\frac{\pi}{2} =$ 0

(f) $\sec\frac{4\pi}{3} =$ Q3 $\frac{1}{\cos\frac{4\pi}{3}} = \frac{1}{-\cos\frac{\pi}{3}} = -2$

(g) $\tan\frac{\pi}{4} =$ Q1 $\frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

(h) $\arccos(\cos\frac{5\pi}{4}) =$ $3\pi/4$, to be in range of arccos

(i) $\arcsin(\sin\frac{5\pi}{6}) =$ $\pi/6$, to be in range of arcsin