Name: Answer Key

Math 1316: Mastery Quiz 8 (Version A)

Please show all your work for computations, and write your final answers in the boxes.

1. Your friend tells you that $\sin(2x) = 2\sin(x)$ for any angle x. When you asked them to justify this, they said that if $x = \pi$ then $\sin(2\pi) = 0$ and $2\sin(\pi) = 0$ so they are the same. Explain why your friend is wrong by giving a different angle which is a counterexample to their claim.

Needs to be true for all angles, not just one. Try $x = \frac{\pi}{2}$:

$\sin(2\cdot\frac{\pi}{2}) = \sin\pi = 0$ but $2\sin(\frac{\pi}{2}) = 2\cdot 1 = 2$, and $0 \neq 2$.

Instead, use double angle identity: $\sin(2x) = 2\sin x\cos x$

2. Find the exact value of $\cos(75°)$. [Hint: $\cos(45°) = \sin(45°) = \sqrt{2}/2$ while $\cos(30°) = \sqrt{3}/2$ and $\sin(30°) = 1/2$.]

$\cos(75°) = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos(75°) = \cos(45° + 30°)$

$= \cos(45°)\cos(30°) - \sin(45°)\sin(30°)$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

3. You are trying to compute the trig functions of an angle, call it β. You ask the local oracle for help. Rather than tell you about β, she tells you about $\alpha = \beta/2$ which is an angle in quadrant 1. Specifically, she tells you that $\sin(\alpha) = 4/5$. Using this information, find $\sin(\beta)$ and $\cos(\beta)$. [Hint: $\beta = 2\alpha$, and you probably want to first calculate $\cos(\alpha)$ before you calculate $\sin(2\alpha)$ and $\cos(2\alpha)$.]

$\sin(\beta) =$ $\frac{24}{25}$
$\cos(\beta) =$ $-\frac{7}{25}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\left(\frac{4}{5}\right)^2 + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - \frac{16}{25} = \frac{9}{25}$

$\cos\alpha = +\frac{3}{5}$

$\beta = 2\alpha$, so use

$\sin\beta = 2\sin\alpha\cos\alpha = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$

$\cos\beta = \cos^2\alpha - \sin^2\alpha = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$

$= \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$