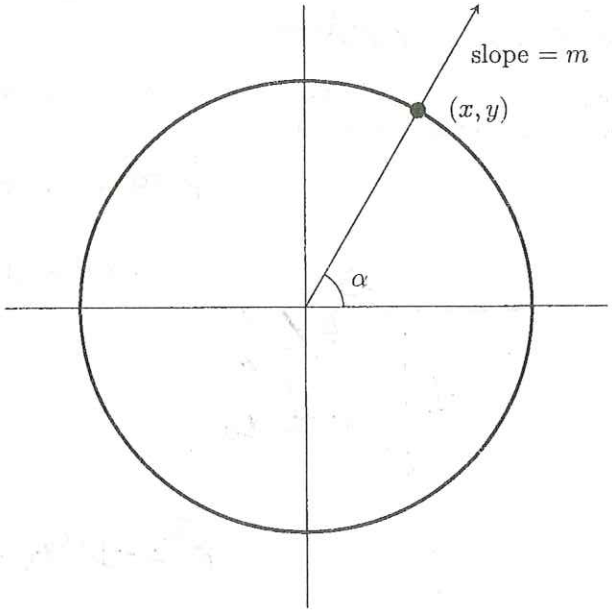
Math 1316: Mastery Quiz 3 (Version A)

September 20, 2022

Name: ANSWER KEY

Please show all your work for all problems, and write your final answers in the boxes.

1. The ray coming out of the origin at angle α intersects the unit circle at a point (x, y) and has a slope m, as in the following diagram. Give the six trig functions of α in terms of x, y, and m. *[Note: There's multiple correct answers for some of these, but I'm only asking you to give one.]*



$\sin\alpha =$ y	$\cos\alpha =$ x	$\tan\alpha =$ $\frac{y}{x}$ or m
$\csc\alpha =$ $\frac{1}{y}$	$\sec\alpha =$ $\frac{1}{x}$	$\cot\alpha =$ $\frac{x}{y}$ or $\frac{1}{m}$

2. The ray going out from the center of the unit circle at an angle θ intersects the circle at the point $(5/13, -12/13)$. Find $\sin(\theta)$, $\sec(\theta)$, and $\tan(\theta)$.

$\sin(\theta) = -\frac{12}{13}$

$\sec(\theta) = \frac{13}{5}$

$\tan(\theta) = -\frac{12}{5}$

these are the x & y coordinates, from which you can compute all trig functions. See the previous question.

$\sec\theta = \frac{1}{\cos\theta} = \frac{13}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-12/13}{5/13} = -\frac{12}{5}$

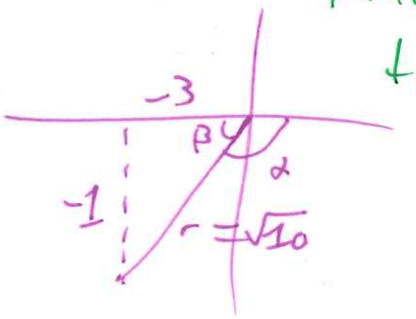
3. An angle α is in Quadrant 3 and has reference angle β. Suppose you know that $\tan\beta = 1/3$. Using this information, compute $\sin(\alpha)$, $\cos(\alpha)$, and $\tan(\alpha)$. **Give exact answers.**

$\sin(\alpha) = -\frac{1}{\sqrt{10}}$

$\cos(\alpha) = -\frac{3}{\sqrt{10}}$

$\tan(\alpha) = \frac{1}{3}$

Draw a reference triangle!



$r^2 = (-1)^2 + (-3)^2$

$= 1 + 9$

$= 10$

$r = \sqrt{10}$

Name: ANSWER KEY

Math 1316: Mastery Quiz 4 (Version A)

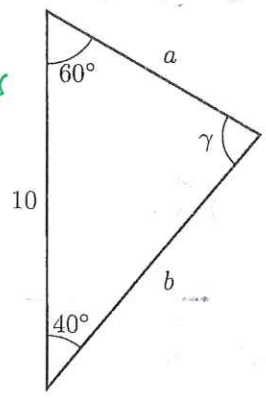
Please show all your work for computations, and write your final answers in the boxes.

1. You have a friend who is a bit of an ass and likes to play pranks. They tell you three of the six values about a triangle, and ask you to compute the remaining three. You suspect they may be trying to trick you by giving you values that don't uniquely identify a triangle. Without doing any calculations using the laws of sines and cosines, how would you check whether what they gave uniquely identifies a triangle? Briefly explain.

this is the most important part. looking at the pattern of data given you can tell whether there's a possibility of >1 solution without doing any calculations

If the info given is AAA or ASS, it could be a problem. AAA is always ambiguous, ASS could have one or two solutions. Trying to solve an ASS triangle with LoC or LoS both can be found, or if one is impossible that will be apparent — LoC or LoS will give an negative side length or angle looking for the triangle.

2. Find all missing angles and sides on the following triangle. Round your answers to two digits past the decimal point, and give angles in degrees.



Need to find γ first.

$\gamma = 180° - 60° - 40°$

$\frac{\sin 40°}{a} = \frac{\sin 80°}{10} \Rightarrow \frac{a}{\sin 40°} = \frac{10}{\sin 80°}$

$a = \frac{10 \sin 40°}{\sin 80°}$

$a \approx 6.53$

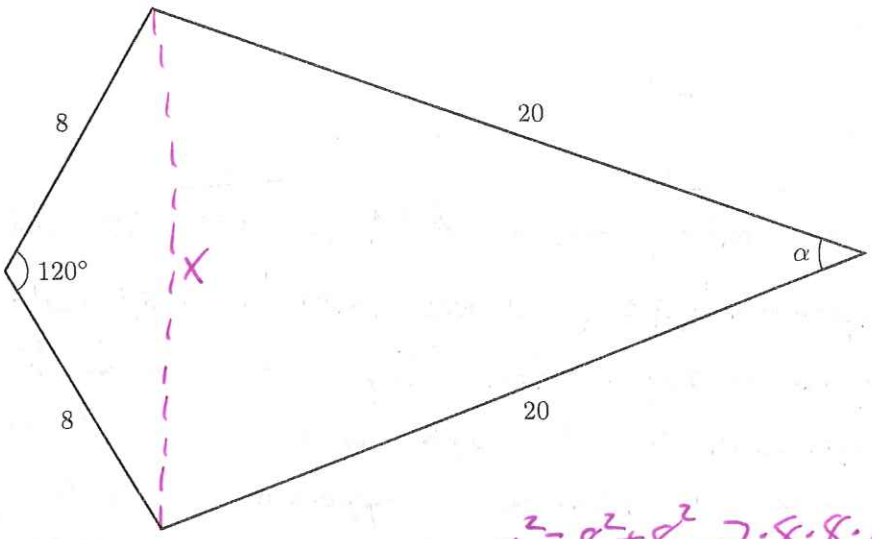
$\frac{\sin 60°}{b} = \frac{\sin 80°}{10} \Rightarrow \frac{b}{\sin 60°} = \frac{10}{\sin 80°}$

$b = \frac{10 \sin 60°}{\sin 80°}$

$b \approx 8.79$

$a =$	6.53
$b =$	8.79
$\gamma =$	80°

3. A kite consists of two sides each of length 8 with a 120° angle between them, and two sides each of length 10, as in the below picture. Find the angle α between the sides of length 10. Round you answer to two digits past the decimal point. [Hint: A quadrilateral is just two triangles joined together.]



$\alpha =$ 40.54°

$x^2 = 8^2 + 8^2 - 2 \cdot 8 \cdot 8 \cdot \cos(120°)$

$= 128 - 128(-\frac{1}{2})$

$= 128 + 64$

$x^2 = 192$

$x = 8\sqrt{3} \approx 13.85641$

$x^2 = 20^2 + 20^2 - 2 \cdot 20 \cdot 20 \cdot \cos\alpha$

$192 = 800 - 800\cos\alpha$

$800\cos\alpha = 800 - 192 = 608$

$\cos\alpha = \frac{608}{800}$

$\alpha = \cos^{-1}\left(\frac{608}{800}\right) \approx 40.54°$

Alternative Approach:

the kite is symmetric, so the horizontal diagonal splits each angle exactly in half

Use Law of Sines: (*)

$\frac{\sin(\frac{\alpha}{2})}{8} = \frac{\sin 60}{20} \Rightarrow \sin(\frac{\alpha}{2}) = \frac{8}{20}\sin 60$

$\sin(\frac{\alpha}{2}) = \frac{8\sqrt{3}}{40} \Rightarrow \frac{\alpha}{2} = \sin^{-1}\left(\frac{8\sqrt{3}}{40}\right)$

$\frac{\alpha}{2} \approx 20.268$

$\alpha = 2 \cdot \frac{\alpha}{2} \approx 40.540$

(*) This is an ASS triangle, so it may be ambiguous. But $20 > 8$, so the second possibility would look like:  which is not a triangle.