Math 1316: 4-26 Worksheet

April 26, 2022

For this worksheet I want you to do some explorations with the double angle and half angle identities. Let me remind you what they say:

$$\begin{aligned}
\sin(2x) &= 2\sin x \cos x \\
\cos(2x) &= \cos^2 x - \sin^2 x \\
&= 2\cos^2 x - 1 \\
&= 1 - 2\sin^2 x \\
\sin(x/2) &= \pm\sqrt{\frac{1-\cos x}{2}} \\
\cos(x/2) &= \pm\sqrt{\frac{1+\cos x}{2}}
\end{aligned}$$

1. If the angle x is in Quadrant 1, what Quadrants can the angle $2x$ be in? Can you determine what Quadrant $2x$ is in, knowing the values of $\sin x$ and $\cos x$, by using the angle sum identities? If so, which one do you use?

2. if the angle x is in Quadrant 2, what Quadrants can the angle $2x$ be in? Can you determine what Quadrant $2x$ is in, knowing the values of $\sin x$ and $\cos x$, by using the angle sum identities? If so, which one do you use?

3. This question is about how you determine whether to use the positive or negative square root for the half angle formulas: If the angle x is in Quadrant 1, what Quadrants can the angle $x/2$ be in? If x is in Quadrant 2, what Quadrants can $x/2$ be in? What if x is in Quadrant 3? Or Quadrant 4?

4. Recall that $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$. Using this and the half angle identities, you can compute $\sin(\pi/8)$ and $\cos(\pi/8)$. [Hint: $\pi/8 = \frac{\pi/4}{2}$.] Once you know $\cos(\pi/8)$ you can again use the half angle identities to compute $\sin(\pi/16)$ and $\cos(\pi/16)$. You can keep dividing by 2, and keep using the half angle identity. Can you recognize the pattern, and give a general answer for what $\sin(\pi/2^N)$ and $\cos(\pi/2^N)$ are?

5. Using the double angle and angle sum identities, can you come up with formulas for $\sin(3x)$, $\cos(3x)$, $\sin(4x)$, and $\cos(4x)$?