Math 1316: 4-21 Worksheet

April 19, 2022

For this worksheet I want you to do some open-ended explorations with the angle sum identities. Let me remind you what they say:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

1. An important concept in calculus is the *difference quotient*. For a function $f(x)$, the difference quotient is the fraction
$$\frac{f(x+h) - f(x)}{h},$$
where x is an arbitrary input and h is a small value. Write the difference quotients for $\sin(x)$ and $\cos(x)$ and use the angle sum identities to simplify them. Use Desmos to graph these difference quotients, based on the parameter h. Look at how these graphs behave when h is approximately 0. Can you find trig functions of x which look like these difference quotients for very small h?

2. The angle sum identities are about adding two angles. Can you come up with variations for adding *three* angles? Think about $\sin(x+y+z)$ as $\sin(x+(y+z))$, and use the double angle identity multiple times. Do you get something different if you instead think of it as $\sin((x+y)+z)$? And ditto for $\cos(x+y+z)$.

3. If you have too much spare time, do the same thing for adding four angles.

4. Knowing the trig values for $30°$, $45°$, and $60°$, you can determine the values for all multiples of $30°$ and $45°$. Using the angle sum formulas you can extend this to computing the values of the trig functions for all multiples of $15°$. Compute trig values for $15°$, $75°$, and $105°$. If you have time, continue this for the other multiples of $15°$.