MATH 321: IN-CLASS WORKSHEET 7
FRIDAY, FEBRUARY 26TH

Today's worksheet asks you to consider a setup similar to the pointing question from lecture, but where the pointing is replaced with something symmetric.

Consider the following setup. You are at a party with a finite number of people, and throughout the course of this party you shake hands with a number of fellow party-goers. Perhaps you shake someone's hand only once, perhaps you shake it multiple times, perhaps you never shake their hand. For each person, let's call the total number of times they shake hands with someone else their *handshake number*. And let's call the whole situation a *handshake party*, so we have a name to refer to it.[1]

(1) Consider an example of a handshake party with four people. Draw a representation of a possible way four people could shake hands with each other. Determine the handshake number of each person, and count up the total number of handshakes. Compare your results to those from your groupmates. Do you notice any patterns?
(2) Do the same thing with five people instead of four people. Compare your results in your group. Do the patterns you observed from before continue to hold?
(3) Prove the following statement: For any handshake party, the sum of the handshake numbers for everyone is twice the total number of handshakes.
(4) Use this to derive the following as a corollary: For any handshake party, there must be an even number of people with an odd handshake number.
(5) Time permitting, show that this corollary is the best possible. That is, show that for any natural number n and even number $k \leq n$ there is a handshake party with n many people in which k people have an odd handshake number. [Hint: Fix an arbitrary even number k and then do induction on $n \geq k$.]
(6) Submit on gradescope your proofs for (3) and (4). If you solved (5) and would like me to look at your proof, you can also submit that. (It is okay to collaborate on the proofs with your groupmates; what you submit should be your own writeup, but you should freely work together and share ideas.)

[1] You can cast all this in purely graph theoretic terms. We are looking at a finite (undirected) *graph*, where each handshake between people is an *edge* between *nodes* and the handshake number is the *degree* of a node.