MATH 321: IN-CLASS WORKSHEET 5
FRIDAY, FEBRUARY 12TH

Today's worksheet is about practicing induction proofs.

As we discussed in lecture, induction is a method of proving a universal statement about natural numbers. Rather than directly prove the statement for an arbitrary n, it is enough to prove the base case and the inductive step. For the inductive step, you show that if the statement is true for n then it is true for $n+1$. For the base case, you directly check the result for your starting point. Usually this is $n = 0$, but if you, for example, are proving something is true for all natural numbers ≥ 5, then $n = 5$ would be your base case.

There is an alternate formulation of induction, what the book calls strong induction, where you get to assume more in the inductive step. Rather than assume the result is true just for the immediate prior natural number, you assume it's true for all smaller numbers. That is, the inductive step is you want to show that if the result is true for all $k < n$, then it must be true for n.

For these problems, I ask you to attempt to prove some theorems yourself before looking at the textbook's proofs. In general, this is a good habit to have when reading mathematics. After you see the statement of a theorem, think for a bit about how you might try to prove it before you read the author's proof. Even if you cannot fully figure out the proof on your own, you'll better understand what's going on if you wrestled with it yourself a bit first.

If you don't have time to get through all of these, that is fine. It's better to take your time to fully understand only some of the proofs, rather than rushing and not having time to understand any of them.

(1) Look at Theorem 23 on page 30 of the textbook, that if you sum up the first n many odd natural numbers you get n^2. Attempt to prove this theorem yourself using induction, before looking at the proof in the book. Does your strategy match the one from the textbook, or did you use a different strategy?
(2) Look at Theorem 24 on the same page, that the sum of the powers of 2 up to 2^n is $2^{n+1} - 1$. Attempt to prove this theorem yourself using induction, before looking at the proof in the book.
(3) Look at Theorem 28 on page 32 of the textbook, that $n^2 < 3^n$ for all natural numbers n. Attempt to prove this theorem yourself using induction, before looking at the proof in the book.
(4) This is a case where you need strong induction, not just common induction. Look at Theorem 30 on page 34 of the textbook, that every natural number has a unique binary representation. Attempt to prove this theorem yourself using strong induction, before looking at the proof in the book.
(5) Submit to gradescope a short paragraph about what you learned with this exercise.