MATH 321: IN-CLASS WORKSHEET 14
FRIDAY, APRIL 23RD

For this exercise I want you to think about using the Cantor–Schröder–Bernstein theorem to prove sets are equinumerous. Recall that this theorem states that if $A \lesssim B \lesssim A$ then $A \simeq B$. That is, to show that there is a bijection between A and B it suffices to construct injective functions in each direction. This is usually much easier than trying to directly construct a bijection.

Some of these we saw already in class, some are part of the homework for this week. If you don't have time to get to all of these by the end of class, that's fine. Except for the first exercise, for all of these the easiest way to show there's a bijection between the two sets is to construct injections in both directions.

(1) Let $2^{\mathbb{N}}$ denote the set of infinite binary sequences. Show that $2^{\mathbb{N}}$ and $\mathcal{P}(\mathbb{N})$ are equinumerous. [Hint: For this one, you can construct a bijection without too much effort, and don't need to go through CSB.]
(2) Let $\mathbb{N}^{\mathbb{N}}$ denote the set of infinite sequences of natural numbers. Show that $2^{\mathbb{N}}$ and $\mathbb{N}^{\mathbb{N}}$ are equinumerous.
(3) Show that $\mathcal{P}(\mathbb{N})$ and $\mathbb{R}$ are equinumerous.
(4) Show that $\mathbb{R}^3$ and $\mathbb{R}$ are equinumerous.
(5) Show that $\mathbb{R}^n$ and $\mathbb{R}$ are equinumerous, where n is a postive integer. [Hint: you can do this directly, constructing injective functions in both directions. Alternatively, Exercise 13.17 in the textbook sketches out how to prove this by induction on n.]
(6) Submit on gradescope a short (one or two paragraph) summary of what you did for this exercise.