MATH 321: IN-CLASS WORKSHEET 13
FRIDAY, APRIL 16TH

(1) Read the Cranks subsection of Section 13.3 from the textbook, on pages 153–154. For each of the three purported proofs that $\mathbb{R}$ is countable, read the textbook's explanation for why they are wrong. Make sure you and your groupmates all understand the errors.

(2) Consider the following two arguments for why Cantor's theorem is wrong:[1]

> "The claim is that there are more real numbers in the range from zero to one than there are natural numbers. To see that this is false simply realize that you don't actually have to write a decimal point to specify the real numbers in this range. Without the decimal point these real numbers just become natural numbers. Can a rational person believe that there are infinite sequences of digits in the form of real numbers but not infinite sequences of digits in the form of natural numbers? The natural numbers are just an infinite sequence of finite numbers. If you believe n is a natural number then you must also believe that $n \cdot 10$ is a natural number. One more digit! There is always one more digit (that is what infinity implies). If there really are an infinite number of natural numbers then some of them must be of a transfinite number of digits or else you would be including numbers in the list more than once."

and

> "There is only one infinity. It means 'repeat'. It is simply the interplay of finite state with process. You can think of it as an 'infinite loop' in programming. To say that one infinity is smaller than another is to deny that the smaller is infinite. Infinite means without bound."

Explain the flaw(s) in these two arguments. Why are they not valid mathematical arguments?

(3) Submit on gradescope a short (one or two paragraph) summary of what you did for this exercise.

[1]Taken from a Hacker News comment. The comment is in response to a Quanta Magazine article about a recent result by Maryanthe Malliaris and Saharon Shelah on the sizes of certain infinite sets.