MATH 302: WEEK 9 WORKSHEET

Last week's worksheet was about simple harmonic motion, motion of an object whose position x obeys the differential equation

$$x'' + \omega^2 x = 0.$$

You saw that simple harmonic motion takes the form of a sine wave, oscillating back and forth between two values forever.

Today's worksheet is about *damped harmonic motion*, where there is a dampening force which slows down the object. Specifically, an object moves according to damped harmonic motion if its position x as a function of time t behaves according to the second-order differential equation

$$x'' + 2rx' + \omega^2 x = 0,$$

where r and ω are positive constants. The coefficient $2r > 0$ represents the resistance force which slows down the particle.

Hereon, consider an object which moves according to damped harmonic motion.

(1) Determine the form of the general solution for x in the *underdamped* ($r^2 < \omega^2$), *overdamped* ($r^2 > \omega^2$), and *critically damped* ($r^2 = \omega^2$) cases. Use computer tools to graph different particular solutions for the three cases. In which cases is the motion *oscillatory*? (Meaning that it repeatedly moves back and forth as time goes to infinity.)

(2) Determine $\lim_{t\to\infty} x(t)$ for each case. Interpret what this means for an object moving according to damped harmonic motion. [Hint: even though the three cases have different formulae for x, the answer is the same in all cases.]