MATH 302: WEEK 5 WORKSHEET

Today's worksheet is about heat transfer. Newton's Law of Cooling states that the rate of heat change of an object is proportional to the difference between its temperature and the ambient temperature. While this Law is not 100% accurate in all circumstances, it is accurate in many cases. (Specifically, the assumption needed is that the *heat transfer coefficient* is constant, or approximately constant.)

Problem. Suppose T is the temperature of an object, which is a function of time t. The object is immersed in a medium with temperature T_0, and the medium is sufficiently large that T_0 remains (approximately) constant even as heat flows between the medium and the object. Newton's Law of Cooling then states that the change in T is proportional to the difference between T and T_0.

(1) Write a differential equation in terms of the variables T and t, and the constant T_0 and a new constant k, which describes the change in temperature.

(2) Solve this differential equation to express T as a function of t. (Your answer should depend on the constants k and T_0, as well as a new constant which comes from solving the differential equation.)

(3) Use your answer to the previous problem to answer: An object with temperature $100°$ is placed in a medium with temperature $40°$. After 1 minute, the object has cooled to $80°$. Assuming that Newton's Law of Cooling applies, how long will it take for the object to cool to $60°$? To $50°$?

(4) What temperature does the object approach as $t \to \infty$?