MATH 302: WEEK 12 WORKSHEET

The previous two worksheets, before spring break, were about free harmonic motion or unforced harmonic motion, motion of an object whose position x obeys the differential equation

$$x'' + 2rx' + \omega^2 x = 0.$$

Today's worksheet is about *forced harmonic motion*, where there is an external force applied to the linear system. This is motion where the position x obeys the differential equation

$$x'' + 2rx' + \omega^2 x = f(t),$$

where $f(t)$ is the *forcing function*. Some disciplines call $f(t)$ the *input* to the system and $x(t)$ the *output* to the system. The idea is, if you apply the force $f(t)$ to the system in produces the outcome of movement $x(t)$.

We specifically will look at forced undamped harmonic motion, where the resistence coefficient r is 0.

(1) Consider an object moving according to forced undamped harmonic motion, i.e. according to the differential equation

$$x'' + \omega^2 x = f(t).$$

Suppose the forcing function is of the form $f(t) = a\sin(\omega_0 t)$, where $\omega_0 \neq \omega$ is a positive constant and a is a positive constant. Solve this differential equation subject to the initial condition $x(0) = 0$ and $x'(0) = b$, where $b > 0$ is a constant. Use this solution to describe the longterm behavior of the motion.

(2) Consider again an object moving according to forced undamped harmonic motion, except this time the forcing function is $f(t) = a\sin(\omega t)$. That is, the forcing function has the same frequency as from the system. Solve this differential equation subject to the initial condition $x(0) = 0$ and $x'(0) = b$, where $b > 0$ is a constant. Use this solution to describe the longterm behavior of the motion.