MATH 302: WEEK 1 WORKSHEET

For this worksheet you will need to use a direction field plotter, such as the one found here: `https://www.geogebra.org/m/W7dAdgqc`

(1) Use the direction field plotter to graph direction fields for differential equations of the form $y' = Ky$ for different constants K. When does the differential equation have an asymptotically stable equilibrium solution as $x \to \infty$, and what value is approached? Building on this, find a differential equation with an asymptotically stable equilibrium solution as $x \to \infty$ which approaches the value 3. And do the same for approaching an arbitrary value v.

(2) Use the direction field plotter to graph direction fields for differential equations of the form $y' = Ky(M - y)$, where K and M are positive constants. These are logistic differential equations, which are used to model population growth given a maximum carrying capacity M. If you have the initial value $y(0)$ is positive, what value is approached as $x \to \infty$? Compare this behavior to what happens with the differential equations $y' = Ky$, and discuss the relative merits/dismerits of the two kinds of differential equations for modeling population growth.

(3) Use the direction field plotter to graph direction fields for differential equations of the form $y' = f(x)$, for the following choices for $f(x)$: $f(x) = e^x$, $f(x) = \cos(x)$, and $f(x) = 1/x$. Use your knowledge of calculus to determine the general solutions for these three differential equations.