MATH 302: STUDY GUIDE FOR MIDTERM 2

Here are the topics you should know for the midterm:

(a) Definitions for linear differential equations, what forms solutions take, and definitions involving linear operators.
(b) How to solve linear homogeneous differential equations.
(c) How to use the methods of undetermined coefficients and of variation of parameters to solve non-homogeneous linear differential equations.
(d) The definition of the Laplace transform and how to compute the Laplace transform of a function.
(e) How to use Laplace transforms to solve a linear differential equation.
(f) How to use series methods to solve and/or approximate solutions to linear differential equations.
(g) How to apply linear differential equations to answer questions about harmonic motion.

Here are some sample problems you can do to test your preparation:

(1) How many linearly independent solutions are there to the differential equation $P(D)y = e^x$, where $P(D)$ is a degree 5 polynomila operator.
(2) Find the general solution to the differential equation
$$y''' + 6y'' + 9y' = 0.$$
(3) Find the general solution to the differential equation
$$y'' + y' + 2y = 2x.$$
(4) Find the particular solution to the differential equation
$$y'' + 2y' - 3y = e^{-t}$$
satisfying the initial conditions $y(0) = y'(0) = 1$.
(5) Consider the function $f(t)$ defined as
$$f(t) = \begin{cases} 2-t & \text{if } 0 \le t \le 2 \\ 0 & \text{otherwise.} \end{cases}$$
Find the Laplace transform $\mathcal{L}[f]$.
(6) Use series methods to find a degree 4 approximation to the power series for the solution to
$$y'' + (x+2)y' - y = e^{2x}$$
satisfying the initial conditions $y(0) = 0$ and $y'(0) = 1$.
(7) Find the general solution to the differential equation
$$y'' + 2y' + y = e^{-x}/x.$$
(8) Suppose a particle's position $y(t)$ moves in damped harmonic motion with oscillation frequency 1π seconds. Write a homogeneous linear differential equation describing the position, in terms of an unknown resistance coefficient r. Consider the following setup. The particle starts at rest, at a distance of 2 meters from its equilibrium position. It is then released and begins to oscillate, with the amplitude decreasing as time goes on. You measure that the amplitude after one period is 1 meter. Determine the resistance coefficient r for this system.