MATH 302: STUDY GUIDE FOR MIDTERM 1

Here are the topics you should know for the midterm:

(a) What it means for something to be a solution to a differential equation, and how to check whether something's a solution.

(b) What it means for a differential equation to be first-order, versus second-order or other higher-order.

(c) Given an n-parameter family of solutions to a differential equation, how to find the particular solution satisfying an initial condition.

(d) How to solve a first-order differential equation, including methods for the following:
  - Separable equations;
  - Equations with homogeneous coefficients;
  - Exact equations;
  - Finding an integrating factor to make an equation exact;
  - Linear equations;
  - Bernoulli equations.

(e) How to use first-order differential equations in applications.

Here are some sample problems you can do to test your preparation:

(1) Find a 1-parameter family of solutions to the equation $(2y - xy \log x) \, dx - 2x \log x \, dy = 0$.

(2) Find the particular solution to the equation $y' + 8x^3y^3 + 2xy = 0$ satisfying the initial condition $y(0) = 1$.

(3) Find a 1-parameter family of solutions to the equation $(x^2y - 1)y' + xy^2 = 1$.

(4) Find the particular solution to the equation $xyy' + x^2 + y^2 = 0$ which satisfies the initial condition $y(2) = 1$.

(5) Find a 1-parameter family of solutions to the equation $y' + 2y = \sin x$.

(6) Find the particular solution to the equation $(x^2+y^2) \, dy + 2x(2x+y) \, dx = 0$ satisfying the initial condition $y(1) = 0$.

(7) Is $t = s^2$ a solution to the equation $(\frac{dt}{ds})^2 = 4t$? Explain why or why not.

(8) Explain why the equation $\log(-y') = y$ has no solution.