Math 302: Series methods, II

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Last week

$$y'' + a(x)y' + b(x)y = c(x)$$

We talked about one method for solving differential equations based on power series.

- If the coefficient functions are analytic at a point p, we can guess that the solution is given by a power series centered at p:

$$y = \sum_{n=0}^{\infty} a_n (x-p)^n.$$

- Plug this into the equation and you can solve for the coefficients a_n.

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- You can determine a recurrence relation which lets you compute a_n in terms of previous values.
- Or you can compute the first few terms to get an approximate solution.

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There are other methods using series to find or approximate solutions, but due to time constraints we won't learn them.

The basic idea is the same: if we guess that the solution is given by a power series we can use that information to allow us to solve for the coefficients.

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For this one, we have to multiply two power series!

An example not centered at 0

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We have to expand the coefficient functions as power series centered on $p = 1$:

- $x = (x-1) + 1$
- $3 = 3$
- $x^2 = (x-1)^2 + 2(x-1) + 1$

Yet another example

$$x^2y'' = x + 1, \qquad y(1) = 0, y'(1) = 1$$