Math 302: Separable differential equations

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Differentials

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- We call dv the differential for the variable v. It is a new variable representing an infinitesimal change in v.

An example

$$\frac{\mathrm{d}y}{\mathrm{d}x}(yx^2 - 2x^2) = y^3$$

Separable differential equations

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If an ODE is separable, it has a family of solutions given by the implicit equation

$$\int P(y)\,\mathrm{d}y = \int Q(x)\,\mathrm{d}x + C,$$

where C is an arbitrary constant.

Why does this work????

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This is an application of the chain rule.

Another example

Let's find a 1-parameter family of solutions for

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = r \tan\theta$$

Yet another example

Let's find a particular solution, satisfying the initial condition $y(0) = 0$, to

$$\frac{\mathrm{d}y}{\mathrm{d}x}(1 - x) = x(y + 1)$$