Math 302: First-order ODEs

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When we've seen examples, it's been clear from context which method to use. E.g. this must be an exact differential equation because we're currently talking about how to solve exact differential equations.

- What should you do if you don't know in advance what method to use?

An example

$$y' - y = e^x$$

Another example

$$(x^2 + y^2)\,\mathrm{d}y + 2xy\,\mathrm{d}x = 0$$

Suggestions on what to do

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1. First try methods which don't need substitution or integrating factors. If you can see that the equation is separable, exact, or linear, then you can immediately use the corresponding method to find the solution.
2. But the equation might not fall into one of these categories, and you might have to do a substitution or find an integrating factor.
   - If the equation has homogeneous coefficients, you can divide by a power of x (or y) and do a substitution to get a separable equation.
   - If $(P_y - Q_x)/Q$ only depends on x, you can find an integrating factor. And similarly if $(Q_x - P_y)/P$ only depends on y.
   - If it is a Bernoulli equation, you can do a substitution to make it linear.

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3. It might be none of these work! You might still be able to find a substitution or integrating factor which transforms your equation into one you can solve. But this is in general a hard problem.

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  - Compare to integration: You can solve $\int e^{-x^2}\,dx$, in the sense that there is some function which is the antiderivative of e^{-x^2}. But this antiderivative cannot be written in terms of elementary functions.

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  - Compare to integration: You can solve $\int e^{-x^2}\,dx$, in the sense that there is some function which is the antiderivative of e^{-x^2}. But this antiderivative cannot be written in terms of elementary functions.
- Sometimes, the best that can be had is a numerical approximation for the solution.
- And visualizing tools like slope fields can help you understand a differential equation, even if you cannot write down an exact solution.