Math 302: Introduction, part II

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These examples suggest a pattern, let's call this a conjecture:

Conjecture

A first-order differential equation has infinitely many solutions, and the general solution is given by one parameter.

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- $(y' - y)(y' - 2y) = 0$

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Consider again the differential equation $y' = Ky$, whose general solution is $y = Ce^{Kx}$. Here, we know that $C = y(0)$.

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- For many applications, the independent variable is time t, so the initial conditions are the state of the system at time $t = 0$.

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Let's find the particular solution satisfying the initial conditions $y(0) = 1$ and $y'(0) = 1$.