# Math 302: Introduction

Kameryn J Williams

University of Hawaiʻi at Mānoa

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# An introductory example

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In symbols:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -Rm \qquad (R > 0)$$

# An introductory example

You have 1000 $g$ of a radioactive substance whose half-life is 2 years. (That is, after 2 years you will have half the mass you started with.) How much of the substance will you have after 5 years?

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- A partial differential equation is a differential equation where you have partial derivatives—one variable is a function of multiple other variables.
- This class focuses on ordinary differential equations, those with two variables where one variable is a function of the other.

In many disciplines, from finance to ecology to physics, quantities can be described by differential equations. We want to be able to solve these differential equations so we can describe the quantities more directly.

# Solutions to differential equations

Consider the ordinary differential equation describing $y$, a variable dependent on the independent variable $x$:

$$\frac{2y}{y'} = x - 1.$$

Let's check that $y = x^2 - x$ is a solution to this equation.

# Solutions to differential equations

In general: suppose you have a differential equation in an independent variable $x$ and a dependent variable $y$.

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- Sometimes, what we can find are implicit solutions, an equation in $x$ and $y$ which gives an implicit function which is a solution.

# Implicit solutions

Let's check that $x^2 + y^2 = r^2$, $r > 0$ is an implicit family of solutions, parameterized by $r$, to the differential equation

$$y' = -\frac{x}{y}.$$

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- We will look a little bit at some techniques for computing approximate solutions to differential equations.
- Along the way we will see a lot of examples of how differential equations are applied in other disciplines.
- And we will see a bit of the theory behind one of the central mathematical questions in the subject: When does a differential equation have a solution, and when does a parameterized family of solutions give the unique general solution?