Math 1410: Worksheet 2

August 27, 2021

Name: _____

1. (a) Consider the following functions:

$$f(x) = \frac{x+1}{x-2}; \qquad g(x) = \sqrt{x+4}; \qquad h(x) = (f+g)(x).$$

Write the domains of these three functions in interval notation.

(b) Sketch a graph of a function whose domain is $(-\infty, 0]$ and whose range is $[-1, 1]$.

(c) Sketch a graph of a function whose domain is $[0, 3]$ and whose range is $[2, 5)$.

(d) Consider the following sets:

$$A = \{x \in \mathbb{R} : 0 \leq x \leq 10 \text{ and } x \text{ is an even integer}\}; \qquad B = (-8, 8).$$

Determine $A \cap B$. How many elements does this set have?

2. (a) Consider the functions $f(t) = \frac{t+1}{t-1}$ and $g(t) = 2t$. What is the average rate of change of $f \cdot g$ along the interval $[2, 4]$.

   (b) Suppose you know the following facts about a function $y(x)$: its domain is $(-\infty, \infty)$, its range is $(0, \infty)$, it is increasing, and $\frac{\mathrm{d}y}{\mathrm{d}x}$ is increasing. Sketch a possible graph of what $y(x)$ looks like.

   (c) Consider the following statement: Given a function f and a constant c, the average rate of change of $c \cdot f$ over an interval $[a, b]$ is c times the average rate of change of f over that same interval. Either give a calculation showing tihs statement is true, or else give a counterexample showing it is false. [Hint: think about the definition of average rate of change.]