Math 1410: Worksheet 12

November 19, 2021

Name: _____

1. Recall the angle-sum identities for sin and cos:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

   (a) Use an angle-sum identity to write the difference quotient for $\sin(x)$ from x to $x+h$ in terms of sin and cos of x and h.

   (b) Use an angle-sum identity to write the difference quotient for $\cos(x)$ from x to $x+h$ in terms of sin and cos of x and h.

   (c) Use the desmos online graphing calculator[1] (or similar tool) to graph these difference quotients as a function of x based on a fixed parameter h. When h is very close to 0, what functions do these graphs look like?

[1] See `https://www.desmos.com/calculator`. If you type in the expressions you got for (a) and (b), it'll give you an "add slider for h" button.

2. Recall the half-angle identities for sin and cos:

$$\sin(\alpha/2) = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos(\alpha/2) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

   (a) Calculate $\sin(\pi/8)$ and $\cos(\pi/8)$.

   (b) Calculate $\sin(\pi/16)$ and $\cos(\pi/16)$.

   (c) Can you generalize this to determine $\sin(\pi/2^n)$ and $\cos(\pi/2^n)$ for positive integers n?