Math 1410: Worksheet 11

November 12, 2021

Name: _____

1. Consider the expression
$$\sin^2(\arctan x).$$

   (a) Draw a right triangle which represents the inverse trig function. Use this to simplify the expression. [Hint: your answer should be a rational expression.]

   (b) Find all roots and asymptotes of the rational expression, as well as its end-behavior.

   (c) Use this information to sketch a graph of $\sin^2(\arctan x)$.

2. Consider the expression
$$E(y) = y^2\sqrt{9 - y^2}.$$

   (a) Use the substitution $y = 3\sin\theta$ to rewrite this expression to an equivalent expression $E'(\theta)$ using trig functions in the new variable θ. [Hint: your answer should be based on $\sin\theta$ and $\cos\theta$.]

   (b) Find the domain of $E(y)$.

   (c) What are the values of θ corresponding to the endpoints of the domain of $E(y)$?[1]

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[1] This process of *trig substitution* is used in calculus to solve certain problems from translating to a difficult y-domain to an easier θ-domain.