Study guide for Math 1410 Midterm 2

October 29, 2021

These are the topics you should know for the secondmidterm.

(a) Polynomial and rational functions. In particular: how to determine where they are positive/negative, and how to sketch graphs.

(b) Exponential functions and logarithms.

(c) Trigonometric functions. In particular: how to sketch graphs, how to simplify trigonometric functions using identities.

(d) Angles in the plane.

(e) For all classes of functions, you should know how to compute the general info about them: domain and range,[1] end behavior, zeroes, asymptotes, difference quotients, images and pre-images, and geometric transformations.

These are the sorts of questions you should know how to solve for the first midterm.

(1) Sketch a graph of one period of $f(x) = 4\sin(2x - \pi/4)$. What are the amplitude and period?

(2) Sketch a graph of $g(x) = 3 - e^{-2x}$, identifying all asymptotes. What are the domain and range of g?

(3) Sketch a graph of $h(x) = 2\cot(x/\pi)$. What is the period?

(4) Sketch a graph of $k(x) = \log_3(x - 4)$. What are the domain and range of k?

(5) Sketch a graph of $j(x) = -5\sec(x + \pi)$. What is the period?

(6) Sketch a graph of

$$\ell(x) = \frac{2x^2(x-2)}{-(x-1)^3(x+3)^2},$$

identifying all zeroes and asymptotes, where $\ell(x)$ is positive or negative, and the end behavior of $\ell(x)$.

(7) Consider the polynomial $p(z) = 2(z-1)^2(z+1)^3$. Determine where $p(z) \geq 0$. Write your answer in interval notation.

(8) Consider the function

$$f(t) = \sqrt{\frac{t^2-1}{t^3+4t^2+4t}}.$$

Determine the domain of f.

(9) Consider the function $P(t) = 2000 + 1000 \cdot 10^{0.01t}$. What is the preimage of $[3000, 4000]$ under $P(t)$? Write your answer in interval notation.

(10) Consider the function $f(x) = 3\cos(3x)$. What is the average rate of change of $f(x)$ from $x = \pi/2$ to $x = 5\pi/6$?

[1] For some polynomial and rational functions you need methods from calculus to compute the range, so you aren't expected to know how to do that.

(11) A line passes through the point $(-1, 3)$ and makes an angle of $\pi/3$ clockwise with the positive y-axis. Write an equation for the line.

(12) Fully simplify the following trigonometric expression.

$$(\sec x \tan x + \csc x \cot x) \cdot \frac{\sin x + \cos x}{\sin^2 x + \cos^2 x}$$