Math 1332: Voting Theory, part I

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Voting

A college philosophy club is voting on which thinker to read next: Butler, Foucault, or Sartre.

	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	B	F	F	S	B	F	F	B	S

Voting

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1st choice	B	F	F	S	B	F	F	B	S
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

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1st choice	B	F	F	S	B	F	F	B	S
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

We can summarize:

# of votes	3	3	2	1	0	0
1st choice	F	B	S	F	B	S
2nd choice	B	S	B	S	F	F
3rd choice	S	F	F	B	S	B

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2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

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# of votes	3	3	2	1	0	0
1st choice	F	B	S	F	B	S
2nd choice	B	S	B	S	F	F
3rd choice	S	F	F	B	S	B

	votes		votes
$F > B$	4	$B > F$	5
$F > S$	4	$B > S$	6
$S > F$	5	$S > B$	3

Some definitions and voting methods

- In an election, majority refers to $>50\%$ of the population.
- And plurality refers to share of the population held by the vote winner.

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- The Condorcet winner in an election is the choice that wins over all other choices in one-versus-one pair-ups.
- A voting method satisfies the Condorcet criterion if it selects the Condorcet winner.

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What the previous example showed is that the plurality method doesn't satisfy the Condorcet criterion.

Is there a problem with the Condercet method?

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	Peter	Flint	Ashley
1st choice	Rock	Scissors	Paper
2nd choice	Scissors	Paper	Rock
3rd choice	Paper	Rock	Scissors

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This vote has no Condorcet winner!

Revisting the philosophy example

	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	**B**	**F**	**F**	**S**	**B**	**F**	**F**	**B**	**S**
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

Revisting the philosophy example

	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	**B**	**F**	**F**	**S**	**B**	**F**	**F**	**B**	**S**
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

Suppose David and Ivan know they're the only club members whose 1st choice is Sartre. Since they know the club uses the plurality method to decide the vote, they decide to change their vote:

Revisting the philosophy example

	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	**B**	**F**	**F**	**S**	**B**	**F**	**F**	**B**	**S**
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

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	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	**B**	**F**	**F**	**B**	**B**	**F**	**F**	**B**	**B**

Revisting the philosophy example

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1st choice	**B**	**F**	**F**	**S**	**B**	**F**	**F**	**B**	**S**
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

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	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	**B**	**F**	**F**	**B**	**B**	**F**	**F**	**B**	**B**

Insincere voting or stragetic voting is when someone casts a ballot against their actual preferences, which certain voting methods incentivize.

Fixing the plurality method to be more fair

A run-off election is one in which there are possibly multiple rounds, where after each round candidates with low votes are eliminated until a candidate acheives a majority.

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With the philosophy club:

- Round 1 Foucault gets 4 votes, Butler gets 3 votes, Sartre gets 2 votes. No one has a majority, so Sartre is eliminated and they revote.

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Having multiple rounds can be time consuming and costly, so here's a way the process can be simulated with a single vote.

Definition (Instant Runoff Voting (IRV))

Voters make a full preference ballot as their vote, listing all options in order. If a choice has a majority of first-choice votes, they win. Else, the choice with fewest first-choice votes is eliminated, and those votes redistributed to the voter's next choice. This repeats until a single choice gets a majority.

IRV and philosophy club

	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	B	F	F	S	B	F	F	B	S
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

IRV and philosophy club

	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	B	F	F	S	B	F	F	B	S
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

- Round 1 No one gets a majority, so Sartre is eliminated.

IRV and philosophy club

	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	B	F	F	S	B	F	F	B	S
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

- Round 1 No one gets a majority, so Sartre is eliminated.
- Round 2 The two Sartre votes are redistributed to their choice, both going to Butler. This gives her 5 votes, which is $\approx 56\% > 50\%$, so she wins the vote.

IRV and philosophy club

	Alice	Barbara	Carlos	David	Eric	Fred	Grace	Holly	Ivan
1st choice	B	F	F	S	B	F	F	B	S
2nd choice	S	B	B	B	S	S	B	S	B
3rd choice	F	S	S	F	F	B	S	F	F

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- Round 2 The two Sartre votes are redistributed to their choice, both going to Butler. This gives her 5 votes, which is $\approx 56\% > 50\%$, so she wins the vote.

The IRV method seems more fair than simple plurality voting, and its about the same complexity to implement, so our philosophy club decides to go with this method.

Have they solved the fair voting problem once and for all, or are problems still lurking...?