SEQUENT RULES

Throughout: Γ is a set of formulae, φ, ψ, and θ are formulae, t is a term, x and y are variables.

Structural Rules

- (Assumption) $\dfrac{}{\Gamma \vdash \varphi}$ when $\varphi \in \Gamma$
- (Antecedent Rule) $\dfrac{\Gamma \vdash \varphi}{\Gamma' \vdash \varphi}$ when $\Gamma \subseteq \Gamma'$
- (Chain) $\dfrac{\Gamma \vdash \varphi \qquad \Gamma, \varphi \vdash \psi}{\Gamma \vdash \psi}$

Methods of Proof

- (Proof by Cases) $\dfrac{\Gamma, \psi \vdash \varphi \qquad \Gamma, \neg\psi \vdash \varphi}{\Gamma \vdash \varphi}$
- (Proof by Contradiction) $\dfrac{\Gamma, \neg\varphi \vdash \psi \qquad \Gamma, \neg\varphi \vdash \psi}{\Gamma \vdash \varphi}$
- (Modus Ponens) $\dfrac{\Gamma \vdash \varphi \Rightarrow \psi \qquad \Gamma \vdash \varphi}{\Gamma \vdash \psi}$
- (Contrapositive) $\dfrac{\Gamma, \varphi \vdash \psi}{\Gamma, \neg\psi \vdash \neg\varphi}$

Connective Rules

- ($\vee$ Antecedent) $\dfrac{\Gamma, \psi \vdash \varphi \qquad \Gamma, \theta \vdash \varphi}{\Gamma, \psi \vee \theta \vdash \varphi}$
- (Left $\vee$ Succedent) $\dfrac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \vee \psi}$
- (Right $\vee$ Succedent) $\dfrac{\Gamma \vdash \varphi}{\Gamma \vdash \psi \vee \varphi}$
- ($\wedge$ Antecedent) $\dfrac{\Gamma, \psi \wedge \theta \vdash \varphi}{\Gamma, \psi, \theta \vdash \varphi}$
- (Left $\vee$ Succedent) $\dfrac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \varphi}$
- (Right $\vee$ Succedent) $\dfrac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \psi}$
- ($\wedge$ Succedent) $\dfrac{\Gamma \vdash \varphi \qquad \Gamma \vdash \psi}{\Gamma \vdash \varphi \wedge \psi}$
- (Double Negation 1) $\dfrac{\Gamma \vdash \varphi}{\Gamma \vdash \neg\neg\varphi}$
- (Double Negation 2) $\dfrac{\Gamma \vdash \neg\neg\varphi}{\Gamma \vdash \varphi}$

Quantifier Rules

- ($\exists$ Succ) $\dfrac{\Gamma \vdash \varphi\frac{t}{x}}{\Gamma \vdash \exists x\varphi}$
- ($\exists$ Ante) $\dfrac{\Gamma, \varphi\frac{y}{x} \vdash \psi}{\Gamma, \exists x\varphi \vdash \psi}$ when y is not free in Γ, φ, ψ
- ($\forall$ Ante) $\dfrac{\Gamma, \varphi\frac{t}{x} \vdash \psi}{\Gamma, \forall x\varphi \vdash \psi}$
- ($\forall$ Succ) $\dfrac{\Gamma \vdash \varphi\frac{y}{x}}{\Gamma \vdash \forall x\varphi}$ when y is not free in Γ, φ

Equality Rules

- ($=$ Refl) $\dfrac{}{\Gamma \vdash t = t}$
- ($=$ Sub) $\dfrac{\Gamma \vdash \varphi\frac{t}{x}}{\Gamma, t = t' \vdash \varphi\frac{t'}{x}}$