MATH455: HOMEWORK 0

KAMERYN J WILLIAMS

Exercise 1. *Show that there is no bijection between* $\mathbb{N}$ *and* $\mathbb{R}$*.*

Solution. It was proven in class on Monday, January 6 that $\mathbb{R}$ is in bijective correspondence with $\mathcal{P}(\mathbb{N})$. Accordingly, it suffices to prove that there is no bijection between $\mathbb{N}$ and $\mathcal{P}(\mathbb{N})$. To this end, consider an arbitrary function $f\colon \mathbb{N} \to \mathcal{P}(\mathbb{N})$. Define $D \subseteq \mathbb{N}$ as $D = \{n \in \mathbb{N} : n \notin f(n)\}$. I claim that D is not in the range of f. To see this, suppose it were the case that $D = f(n)$ for some n. If $n \in D$, then by the definition of D we would have that $n \notin f(n) = D$, a contradiction. On the other hand, if $n \notin D = f(n)$, then again by by the definition of D we would have that $n \in D$. Either way, we get a contradiction. So it must be that our arbitrary f is not surjective, and hence fails to be bijective. $\square$

Date: January 10, 2019.