MATH455 HOMEWORK 8
DUE FRIDAY, APRIL 10

Do Exercise 1 and your choice of one of Exercise 2, 3, or 4.

Exercise 1. Suppose that $R(x, y) \subseteq \mathbb{N}^2$ is a computable relation. Prove that $\{(x, z) \in \mathbb{N}^3 : \exists y < z\ R(x, y)\}$ is also computable.

Exercise 2. Prove Smullyan's Double Recursion Theorem: For any computable total functions $f(x, y), g(x, y) : \mathbb{N}^2 \to \mathbb{N}$ there are natural numbers a and b so that $\varphi_a = \varphi_{f(a,b)}$ and $\varphi_b = \varphi_{g(a,b)}$. [See Exercise 3.15(a) on page 39 of Soare for a hint.]

Exercise 3. Show that for any computable total function $f(x, y) : \mathbb{N}^2 \to \mathbb{N}$ there is $n \in \mathbb{N}$ so that for all y we have $\varphi_n(y) = f(n, y)$. [Hint: Use the Kleene Recursion Theorem.]

Exercise 4. A *quine* is a computer program which outputs its own source code and then halts. Write a quine in your favorite programming lannguage. [Hint: use the Kleene Recursion Theorem. Write a program P which outputs the source code for a program Q so that Q outputs the source code for Q.]