MATH455 HOMEWORK 7
DUE FRIDAY, APRIL 3

Do Exercise 1 and your choice of one of Exercise 2, 3, or 4.

Exercise 1. Let H' be the set of all pairs $(P, \vec{R})$ of register machine programs P and input $\vec{R}$ so that P halts with $\vec{R}$ as input. Argue directly that H' is not decidable. (That is, do not derive this as a corollary of the version of the halting problem from blecture. Instead, use a similar diagonal argument to show that H' is not decidable.)

Exercise 2. Consider the following "proof" that there is no computable listing of register machine programs:

- Suppose toward a contradiction that there is a computable listing of register machine programs so that each register machine is assigned an index e so that from e we can compute the corresponding program and vice versa. Now define a register machine program B, taking input a natural number e. Running B on input e does the following: First as a subroutine B runs the e-th register machine program on input e, getting some output $y \in \mathbb{N}$. This can be done similar to how we defined a universal register machine. Then B takes this subroutine output y outputs $y + 1$.

  Fix the index e for the program B. Consider what happens when B is run with e as input. It simulates running B with input e, producing some output $y \in \mathbb{N}$. But by the definition of B we get that it outputs $y + 1$. Since both y and $y + 1$ are the output to B with the same input e we get that $y = y + 1$, which is impossible. This contradiction shows that our original assumption that there is a computable listing of register machine programs must be false.

Identify the flaw in this "proof" and explain why it is not valid.

Exercise 3. Prove that for any natural number e there are infinitely many natural numbers e' so that $\varphi_e = \varphi_{e'}$.[1]

Exercise 4. Prove that for each natural number s the set $\{P : P \text{ is a register machine program which halts in } \leq s \text{ many steps}\}$ is decidable.

[1]Recall from the video for Friday's blecture that φ_e is the function computed by the e-th register machine program, according to some fixed computable listing of the register machine programs.