MATH455 HOMEWORK 2
DUE FRIDAY, FEBRUARY 7

Exercise 1. Do Exercise 4.11 (page 38) from the textbook.

Exercise 2. Write sentences in the language with only logical symbols which express that a structure has at most n elements, has exactly n elements, and has at least n elements, for $n \in \mathbb{N}$. That is, come up with sentences μ_n, ε_n, and λ_n so that if $\mathcal{M}$ is a structure:

- $\mathcal{M} \models \mu_n$ iff $|M| \geq n$;
- $\mathcal{M} \models \varepsilon_n$ iff $|M| = n$; and
- $\mathcal{M} \models \lambda_n$ iff $|M| \geq n$.

Let Graph be the theory in the language with a single binary relation E with the following two axioms:

$$\forall x\ \neg x\ E\ x$$
$$\forall x \forall y\ x\ E\ y \Rightarrow y\ E\ x.$$

A *graph* is a structure $\mathcal{G} = (G, E)$ satisfying Graph.

Recall that a *clique* in a graph $\mathcal{G}$ is a subset $C \subseteq G$ so that $|C| \geq 2$ and for all $x \neq y \in C$ we have $x\ E\ y$. An *anti-clique* is $C \subseteq G$ so that $|C| \geq 2$ and for all $x, y \in C$ we have $\neg x\ E\ y$.

Exercise 3. Write down sentences in the language of graphs which express "there is a clique of size n" and "there is an anti-clique of size n", for each $n \in \mathbb{N}$ with $n \geq 2$.

Exercise 4. Can you come up with a single sentence φ so that a graph $\mathcal{G} \models \varphi$ iff $\mathcal{G}$ has no cliques of any size? If yes, provide the sentence. If no, explain why. What about for anti-cliques?