MATH455 HOMEWORK 0
DUE FRIDAY, JANUARY 24

Recall the truth tables for the following logical connectives.

p	q	$p \wedge q$
t	t	t
t	f	f
f	t	f
f	f	f

p	q	$p \vee q$
t	t	t
t	f	t
f	t	t
f	f	f

p	$\neg p$
t	f
f	t

Definition. *A set S of logical connectives is* universal *if for any finite n and any function f : $\{t, f\}^n \to \{t, f\}$ there is an expression $E(p_1, \ldots, p_n)$ using the propositional variables $p_1, \ldots p_n$ and connectives from S so that $E(p_1, \ldots, p_n) = f(p_1, \ldots, p_n)$ for any assignment of truth values to the propositional variables.*

Exercise 1. Show that $\{\wedge, \vee, \neg\}$ is universal.

Exercise 2. Which of $\{\wedge, \vee\}$, $\{\wedge, \neg\}$, and $\{\vee, \neg\}$ are universal? Justify your answers.

Exercise 3. Consider the following logical connective, defined according to the following truth table.

p	q	$p \uparrow q$
t	t	f
t	f	t
f	t	t
f	f	t

Show that $\{\uparrow\}$ is universal. [Hint: by Exercise 1 it is enough to show that $p \wedge q$, $p \vee q$, and $\neg p$ can all be expressed just using $\uparrow$. (Why?)]

Exercise 4. Consider the following logical connective, defined according to the following truth table.

p	q	$p \downarrow q$
t	t	f
t	f	f
f	t	f
f	f	t

Show that $\{\downarrow\}$ is universal.