Study guide for Math 372 Midterm 2

April 6, 2020

These are the sorts of questions you should know how to solve for the first midterm.

1. A random variable X is normally distributed with mean 11.7 and standard deviation 4.2. Determine $P(X \geq 7.3)$ and $P(7.3 \leq X \leq 14.1)$.

2. Suppose X and Y are random variables satisfying the following.

   - $P(0 \leq X \leq 2) = 1/2$.
   - $P(0 \leq Y \leq 1) = 1/3$.
   - $P(0 \leq X \leq 2 \text{ and } 0 \leq Y \leq 1) = 1/4$.

   Are X and Y independent? Explain why/why not, or explain why there is not enough information to determine.

3. Suppose $X_1, \ldots, X_n$ are a random sample from a distribution you know to be uniform. Is the sample mean $\bar{X}$ an unbiased estimator for the median $\tilde{\mu}$? Explain why/why not, or explain why there is not enough information to determine.

4. Suppose $X_1, \ldots, X_{100}$ is a random sample from a distribution with mean 7.4 and standard deviation 1.1. What is $E[\bar{X}]$? Use the central limit theorem to estimate $V[\bar{X}]$ and $P(\bar{X} > 7.5)$.

5. Suppose X and Y are independent, normally distributed random variables with $X \sim \text{Norm}(10, 2)$ and $Y \sim \text{Norm}(6, 3)$. Determine the following.

   (a) $E[3X - 5Y]$.
   (b) $V[3X - 5Y]$.
   (c) $P(3X + 5Y < 0)$

6. See the file `bigsample.csv` for a random sample ($n = 400$) from a distribution. Determine (approximate) 95% and 99% confidence intervals for the mean μ of the distribution.

7. See the file `normalsample.csv` for a random sample ($n = 9$) from a normal distribution. Determine the 95% t confidence interval for the mean μ of the distribution and the 95% t prediction interval for X_{10}.