Study guide for Math 372 Midterm 1

February 14, 2020

These are the sorts of questions you should know how to solve for the first midterm.

1. Let $\mathcal{S} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ be a sample space and let $A = \{1, 3, 5, 8\}$, $B = \{2, 4, 5, 6\}$, $C = \{1, 2, 3, 5\}$. Assume each simple event has equal probability $P(\{n\}) = 1/8$.
   - Calculate $P(A \cap B)$ and $P(B \cup C)$.
   - Calculate $P(A|C)$ and $P(C|B)$.
   - Are A and B independent? Justify your answer. What about B and C?
2. You shuffle a standard 52 card deck of cards, so that any order of the cards is equally likely, then draw 4 cards. How many different ways are there to make that draw, where you care about the order? What if you don't care about the order? How likely is it that your 4 cards contain only face cards (Jack, Queen, King, or Ace, of which there are four of each in the deck)?
3. You are attempting to navigate a labyrinth. At junctions in the maze there are guides who will tell you which path to take. 60% of the guides are knights, who answer truthfully 90% of the time. The remaining 40% are knaves, who lie 90% of the time. You are at a branching path, with the options to go left or right. You know that the left path will take you to the exit, but you ask the guide nonetheless. She tells you that you should go left. Assuming each individual guide is equally likely to be the one at this junction, what is the probability that this guide is a knave?
4. Consider the discrete random variable X with probability distribution function $f(y)$ given by: $f(1) = 1/10$, $f(2) = 2/10)$, $f(3) = 3/10$, and $f(4) = 4/10$, and $f(y) = 0$ for all other values.
   - Calculate $P(1 \leq X \leq 3)$.
   - Calculate $E[X]$ and $V[X]$.
   - Calculate $E[X^2 + 1]$.
5. Let $X \sim \text{Bin}(4, 2/3)$ be a binomial random variable with parameters 4 and 2/3.
   - Calculate $E[X]$ and $V[X]$.
   - Calculate $P(X \leq 3)$.
   - Calculate $P(X > E[X])$.
6. Let $X \sim \text{Pois}(3)$ be a Poisson random variable with parameter 3.
   - Calculate $E[X]$ and $V[X]$.
   - Calculate $P(X > E[X])$.
7. Let X be a continuous random variable with the following probability distribution function $f(y)$:

$$f(y) = \begin{cases} \frac{e^y}{e-1} & \text{if } 0 \leq y \leq 1 \\ 0 & \text{else.} \end{cases}$$

   - Verify that this really is a probability distribution function. [Hint: you have two things to check.]

- Set up the integrals to calculate $E[X]$ and $V[X]$.
- Evaluate those integrals.

8. Let $X \sim \text{Unif}(5, 25)$ be a uniform continuous random variable with parameters 5 and 25.

   - Calculate $E[X]$ and $V[X]$.
   - Set up the integral to calculate $P(|X - E[X]| < \sigma_X)$.
   - Evaluate this integral.

9. Let $X \sim \text{Norm}(4, 10)$ be a normal random variable with parameters 4 and 10.

   - Sketch a graph of the probability distribution function for X. Mark where the maximum and inflection points of the graph are.
   - Determine which of the following two quantities is larger, or if they are equal: $P(X < 4)$ or $P(X > 10)$.
   - Determine which of the following two quantities is larger, or if they are equal: $P(X > 0)$ or $P(X < 8)$.
   - Determine which of the following two quantities is larger, or if they are equal: $P(X > 14)$ or $P(X < -16)$.