Study guide for Math 372 Final Exam

April 29, 2020

These are the sorts of questions you should know how to solve for the final exam.

1. Let $\mathcal{S} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ be a sample space and let $A = \{1, 3, 5, 8\}$, $B = \{2, 4, 5, 6\}$, $C = \{1, 2, 3, 5\}$. Assume each simple event has equal probability $P(\{n\}) = 1/8$. Let X be the random variable defined as $X(s) = 2s + 1$ and Y be the random variable defined as $Y(s) = 6$ if $s \in A$ and $Y(s) = 0$ otherwise.
   - Calculate $P(A \cap B)$ and $P(B \cup C)$.
   - Calculate $P(A|C)$ and $P(C|B)$.
   - Are A and B independent events? Justify your answer. What about B and C?
   - List all the elements of the event $Y < X$.
   - Calculate $P(X > 10)$ and $P(X > Y)$.
   - Are X and Y independent random variables? Justify your answer.
2. You shuffle a standard 52 card deck of cards, so that any order of the cards is equally likely, then draw 4 cards. How many different ways are there to make that draw, where you care about the order? What if you don't care about the order? How likely is it that your 4 cards contain only face cards (Jack, Queen, King, or Ace, of which there are four of each in the deck)?
3. Consider the discrete random variable X with probability distribution function $f(y)$ given by: $f(1) = 1/10$, $f(2) = 2/10)$, $f(3) = 3/10$, and $f(4) = 4/10$, and $f(y) = 0$ for all other values.
   - Calculate $P(1 \leq X \leq 3)$.
   - Calculate $E[X]$ and $V[X]$.
   - Calculate $E[X^2 + 1]$.
4. Let $X \sim \text{Bin}(4, 2/3)$ be a binomial random variable with parameters 4 and 2/3.
   - Calculate $E[X]$ and $V[X]$.
   - Calculate $P(X \leq 3)$.
   - Calculate $P(X > E[X])$.
5. Let $X \sim \text{Pois}(3)$ be a Poisson random variable with parameter 3.
   - Calculate $E[X]$ and $V[X]$.
   - Calculate $P(X > E[X])$.
6. Let X be a continuous random variable with the following probability distribution function $f(y)$:

$$f(y) = \begin{cases} \frac{e^y}{e-1} & \text{if } 0 \leq y \leq 1 \\ 0 & \text{else.} \end{cases}$$

   - Verify that this really is a probability distribution function. [Hint: you have two things to check.]
   - Set up the integrals to calculate $E[X]$ and $V[X]$.

- Evaluate those integrals.

7. Let $X \sim \text{Unif}(5, 25)$ be a uniform continuous random variable with parameters 5 and 25.
   - Calculate $E[X]$ and $V[X]$.
   - Set up the integral to calculate $P(|X - E[X]| < \sigma_X)$.
   - Evaluate this integral.

8. Let $X \sim \text{Norm}(4, 10)$ be a normal random variable with parameters 4 and 10.
   - Sketch a graph of the probability distribution function for X. Mark where the maximum and inflection points of the graph are.
   - Calculate $P(X > 10)$ and $P(0 \leq X \leq 8)$.

9. Suppose $X \sim \text{Norm}(1, 2)$ and $Y \sim \text{Norm}(-2, 1)$. Determine $E[2X + Y]$ and $V[2X + Y]$. Calculate $P(2X + Y > 1)$.

10. Let $X_1, \ldots, X_{81}$ be a random sample from a normal distribution with mean 10 and standard deviation 18. Calculate $E[\bar{X}]$ and $V[\bar{X}]$. Use the central limit theorem to estimate $P(X > 0)$.

11. Use R (or your favorite other program) to generate a list of 100 randomly generated numbers.[1] Calculate a 95% confidence interval for the mean. Is the true mean in your confidence interval?

12. Use R (or your favorite other program) to generate a list of 10 randomly generated numbers, taken from a normal distribution. Calculate a 99% t confidence interval for the mean and a 99% χ^2 confidence interval for the variance.

13. Let μ denote the true mean for grades in the first semester of calculus. You are told that $\mu \leq 70$, but you want to reject this null hypothesis in favor of the alternative hypothesis $\mu > 70$. To get evidence for this you look at the grades in one class. You calculate that, in the class of 60 students, that their average grade is $\bar{x} = 73$ with a sample standard devation $s = 16$. Use a z test to determine the P-value for this test. If you use a significance level $\alpha = 1\%$, do you reject the null hypothesis?

14. Consider the following table of pairs of values drawn from two distributions.

i	x_i	y_i
1	99.0	28.8
2	101.1	27.9
3	102.7	27.0
4	103.0	25.2
5	105.4	22.8
6	107.0	21.5

Graph a scatterplot of the data, and obtain the least squares regression line. Use this line to predict the value for y when $x = 110$.

[1] In R the command `rnorm(n,mu,sigma)` will return a list of n independent random samples from a normal random variable with mean mu and standard deviation sigma. And there are similar commands for other distributions, e.g. `runif` for uniform distributions.